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The Route to Atomic and Quantum Standards

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For the immediate future, it is already clear that the most advanced clocks will provide interesting new scientific avenues to study our universe, pushing the limits on tests of the most fundamental physical laws to new levels. This includes tests of general relativity and searches for violations of the isotropy of space or a preferred reference frame. Fundamental symmetries between matter and antimatter could be investigated through the comparison of optical clocks, as has been proposed for the 1s-2s transitions in both hydrogen and anti-hydrogen (*51*). To date, optical and microwave frequency standards have already been used in some of the most accurate determinations of the fine structure constant α and the Rydberg constant R_∞ [e.g., (*52*, *53*)], and laboratory comparisons of clocks based on different atomic transitions are now providing some of the most stringent constraints of the possible variation of fundamental constants (*54*, *55*). An example of this kind of experiment is given in Fig. 5A, which shows the measurement of the Hg^+ optical clock transition at $\sim$1064 THz (282 nm) in terms of the cesium hyperfine splitting as realized by NIST-F1 (*54*). Over a period of $\sim$3 years, there is no measurable divergence in the ratio of the output frequencies of these two clocks, constraining a fractional variation of $g_{Cs}(m_e/m_p)\alpha^6$ to be less than 7×10^{-15} per year, where m_e/m_p is the electron-to-proton mass ratio and g_{Cs} is the ^{133}Cs nuclear g factor. Assuming any variation comes only from the α^6 factor, the data constrain any possible linear fractional variation of α to be less than 1.2×10^{-15} per year. The combination of these data with other recent clock comparison experiments has resulted in similar constraints being placed on other fundamental constants, as summarized in Fig. 5B (*56*, *57*).

It seems clear that future atomic clocks will continue to subdivide the second into still smaller units of time. But in contrast to Mermin's original concern, it is more likely that the femtosecond, attosecond (10^{-18} s), or zeptosecond (10^{-21} s) will be considered utterly alien impositions on our macroscopic world, while nonetheless proving to be useful bookkeeping units in the continued quest to better understand the inner workings of the microscopic world.

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REVIEW

The Route to Atomic and Quantum Standards

Jeff Flowers

Over the past half-century, there has been a shift away from standards based on particular artifacts toward those based on physical effects, the most stable being based on quantum properties of systems. This change was proposed at the end of the 19th century but is still not complete at the start of the 21st. We discuss how this vision has been implemented through recent advances in science and metrology and how these may soon lead to an SI system finally free from artifact standards, with a consistency based on fundamental constants.

Quantities, Units, and Standards

To investigate any physical phenomena, we must make measurements, communicate them to others, and record them in a way that will be understandable in the future. To do so, a system of quantities and units is required. Measurement is a comparison process in which the value of a quantity is expressed as the product of a value and a unit; that is,

$$\text{Quantity} = \{\text{numerical value}\} \times [\text{unit}] \quad (1)$$

where the unit is an agreed-upon value of a quantity of the same type. The concept of a quantity such as length is independent of the associated unit; the length is the same whether it is measured in feet or meters. A standard is a

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physical realization of the definition, with an agreed-upon value to be used as a reference.

Systems of measurement have been developed since ancient times. During the 19th and 20th centuries, the development of global trade in goods and international communication between scientists led to the development of an internationally agreed-upon system of units based on standards endorsed by international treaty. This has virtually replaced, especially in scientific and technical use, the confusing multiplicity of units and standards that existed beforehand and were often only in local use (*1*). Over time, as understanding and technology developed, this system was enhanced and developed into the Système International d'Unites (SI), formalized in 1960, that is now in global use. The history leading to the establishment of the SI and changes made by decisions of the Conférence Générale des Poids et Mesures (CGPM), the decision-making body, are recorded in the SI brochure, now in its 7th edition (*2*, *3*).

Early Proposals for Units Based on Physical Phenomena

Early "natural" unit systems used Earth's size and rotation rate and the density of water as units, but these are difficult to realize, so for practical purposes the international standards of mass and length were originally artifacts preserved in vaults. Later, modern artifacts, standard cells, and standard resistors, for example, were invented. It was recognized early on (as in the quotation of Maxwell reproduced below) that a natural system of standards could be based on atomic phenomena that were reproducible and constant and not subject to the vagaries of man and the environment.

> *The earth has been measured as a basis for a permanent standard of length, and every property of metals has been investigated to guard against any alteration of the material standards when made. To weigh or measure any thing with modern accuracy, requires a course of experiment and calculation in which almost every branch of physics and mathematics is brought into requisition. Yet, after all, the dimensions of our earth and its time of rotation, though, relatively to our present means of comparison, very permanent, are not so by any physical necessity. The earth might contract by cooling, or it might be enlarged by a layer of meteorites falling on it, or its rate of revolution might slowly slacken, and yet it would continue to be as much a planet as before. But a molecule, say of hydrogen, if either its mass or its time of vibration were to be altered in the least, would no longer be a molecule of hydrogen. If, then, we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wave-length, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.*
>
> —James Clerk Maxwell, 1890 (*4*), p. 225

SI is a practical system, based on atomic or quantum standards where appropriate, but based on artifacts where utility dictates. It cannot be led by philosophy beyond the bounds of present-day science and technology. The change from artifact to atomic or quantum standards has been a guiding principle, but improved realization and stability are prerequisites of advances.

Role of the Fundamental Constants in Unit Systems

It is common in theoretical work to use the fundamental constants of physics as units. This is often seen in the form of an expression such as "let $\hbar = e = c = 1$" or "let $e = m_e = \hbar = 1$" at the start of a paper. These expressions define an alternative unit system with different base quantities from those of the SI. They could be written as $\{e\} = 1$, and so on, using the notation of Eq. 1, to show the meaning; for example, $e = 1[e]$, the value of the electron charge e, is one e unit. Nowadays, the metrologist is aiming to do in practice what the theoretician is doing in principle, basing the system of units on fundamental constants or atomic systems. These are invariant both on a practical scale and as far as can be measured in the laboratory (*5*, *6*). The possible variability of the fundamental constants currently has a high profile, following evidence for variation in the fine-structure constant over cosmological time (*7*), although this is now disputed (*8*). Variations of the constants would imply new physics that would require changes in the basis of our unit system but would not impact practical measurement at current accuracies. For comparison of a theory with experiment, the conversion must be made to practical (preferably SI) units, and this requires values for the appropriate fundamental constants in SI units at the requisite accuracy.

Length, Time, and Frequency

Mass, length, and time are the quantities that have been measured since earliest times, and they were the basis of early unit systems. We will defer the discussion of mass until later in this article because of the present link with electrical metrology, which we discuss below.

Because of its unrivaled accuracy, frequency measurement is key in many aspects of modern metrology. Early clocks were mechanical, with a pendulum as a frequency standard; the practical change to an atomic system came in 1955 with the cesium clock developed by Essen (*9*) (Fig. 1A). By 1968, further refinement and testing led to the redefinition of the second from one based on the rotation of Earth to an atomic one: "The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom" [13th CGPM, 1967 (*10*)].

The time lag in adopting the atomic standard as the definition reflects the work that is necessary to give confidence in the superiority of a new method and in ensuring that the new value adopted is as close as possible to the old one.

Although the cesium frequency cannot presently be explicitly written in terms of fundamental constants because of the complexity of the atomic theory required, it is a quantum system that will have the stability associated with fundamental constants. The uncertainty in calculating this frequency is many orders of magnitude away from its measurement uncertainty. The Rydberg constant could be considered the natural fundamental constant–based unit of frequency. It is determined with a relative uncertainty of 6.6×10^{-12}, which is currently the limit at which an atomic frequency can be calculated from fundamental constants (*11*).

The choice of the cesium definition was a good one in the sense that the technology, although superior to the alternative clocks of the day, still had much room for improvement, and the definition has endured to this day, during which time its practical realization has improved by five orders of magnitude. However, the time is approaching when optical frequency standards will have accuracies and stabilities superior to the best microwave cesium standards. Then it will be necessary to revisit the definition of the second. There are a number of candidate optical frequency standards (*12*), but at present no particular standard is clearly superior to the others.

The development of modern frequency metrology has led to a measurement capability of astounding accuracy. As it is expanded on in another article in this issue (*12*), I will consider here in detail only one aspect: the change in status of length and the meter.

The speed-of-light definition of the meter. A clear example of the link between fundamental constants and the units is the adoption of the speed-of-light definition of the meter. The meter was originally defined as the length of a prototype meter bar intended to be 1/10,000,000 of the length of a quadrant of Earth. By 1960, the development of interferometry allowed an atomic redefinition of the meter in terms of the wavelength of light from a specific source, the krypton lamp. With the invention of the laser, length measurement by interferometry was radically improved and the krypton standard was not accurate enough. The meter definition could then have been revised using the wavelength of a specified stabilized laser. However, the progress in understanding the metrological importance of the speed of light, along with the progress in its accurate measurements (Fig. 1B), led to the change from defining the meter in terms of the wavelength of light from a specific source, to a fundamental constant–based definition in which the speed of light is a defined quantity (*13*). Thus, the definition is now: "The metre is the length of path travelled by light in vacuum during a time interval of 1/299 792 458 of a second" [17th CGPM, 1983 (*14*)].

The choice of the speed-of-light definition over the use of a particular stabilized laser should ensure that this definition will endure, whereas the krypton definition lasted only 23 years. In practice, a number of "recommended radiations," that is, frequencies of particular stabilized lasers, are published accompanying the definition. This means that to realize the meter to a given uncertainty, it is not necessary to remeasure the frequency of the stabilized laser used.

The differences between the definition of a unit and its realization and practical implementation need to be made clear. To realize the meter, there is no need to measure the distance that light travels in 1/299,792,458 of a second by literally timing a light beam. One can, for example, continue to use a laser interferometer and measure the frequency of the laser used, or use a recommended stabilized laser and then use the relationship $c = f\lambda$ (as well as corrections for refractive index, if the measurement is not done in a vacuum). The realization is a method that implements the definition by using the known laws of physics; it allows the experimental production of a known quantity of the same kind as the one defined, but the method used may be dissimilar to the one in the definition.

Development of Electrical Units

The ampere. The earliest standard to be maintained by means of a fundamental constant was the 1906 international ampere. This standard defined the ampere as current required to deposit 1.18800 mg s^{-1} of silver by electrolysis and hence based the ampere on the Faraday constant, which is given by

$$F = \frac{ItM}{zm} \qquad (2)$$

where m is the mass of a substance of molar mass M and valence z deposited by electrolysis using a current I for time t. This ampere definition endured until 1948, the decision to make a change having been postponed by World War II. It was recognized in the 1930s that there were difficulties with the practical implementation; in practice, the ampere was maintained by using standard cells and resistors to maintain the volt and ohm. This situation was resolved by a change in the definition to the modern one: "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length" [9th CGPM, 1948 (*15*)].

This definition is based on the laws of electromagnetism and combined with the expression for the force F per length l on two parallel infinite wires, each carrying a current I, a distance d apart in a vacuum, namely

$$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi d} \qquad (3)$$

has the effect of assigning to the value of the magnetic constant μ_0 (also known as the permeability of free space), an exact fixed value of $4\pi \times 10^{-7}$ H m^{-1}. The fixed value of the speed of light means that this definition is equivalent to fixing the value of the electric constant ε_0 (also known as the permittivity of free space) and of the characteristic impedance of free space, through the relationships

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \text{ and } Z_0 = \sqrt{\mu_0/\varepsilon_0} \qquad (4)$$

Electrical standards are in practice necessarily derived from mechanical ones. Realization of the ampere by measuring the forces on current-carrying wires is now no longer undertaken, as these experiments were limited at about one part in 10^6 by difficulties in establishing the geometry of the current flow in relation to the wire. Today, the ampere is realized through a combination of realizations of the farad, volt, and watt discussed below.

Absolute realizations of the electrical units. Modern realizations of the electrical units are based on a number of experimental systems that link mechanical and electrical measurement. The ohm can be determined using the calculable capacitor. This is a capacitor whose geometry is such that its capacitance can be accurately calculated using a theorem published in 1956 by Thompson and Lampard (*16*). They found that the cross capacitance of four infinitely long right cylindrical conductors is given by

$$\frac{dC}{dz} = \frac{\varepsilon_0}{\pi}\ln(2) = 1.953\ldots\text{pF m}^{-1} \qquad (5)$$

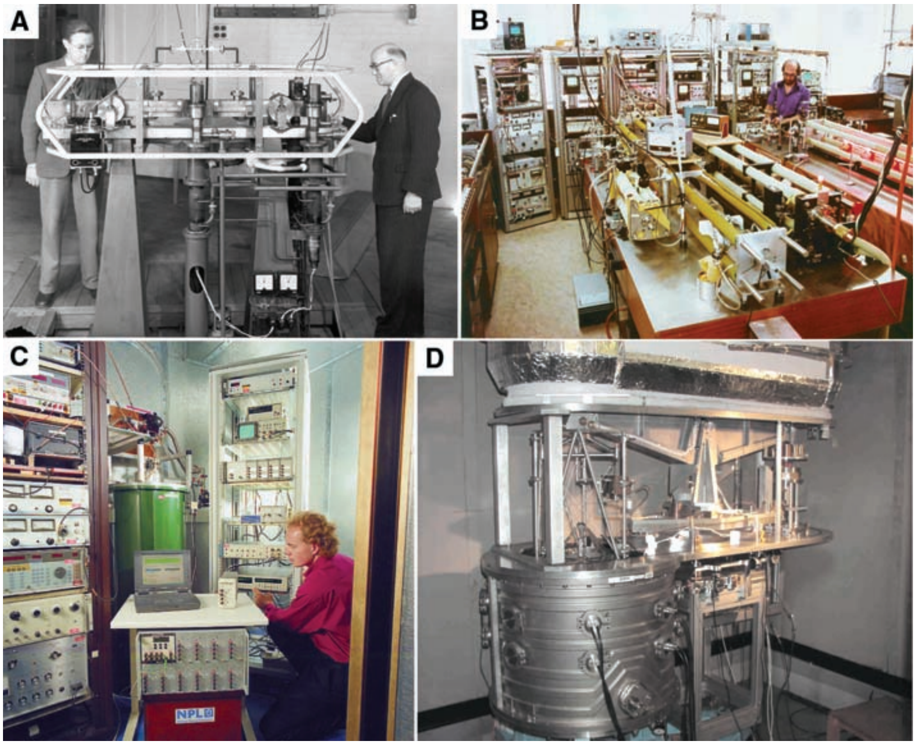

Fig. 1. Some milestones in the development of quantum metrology. (**A**) The cesium beam clock of Essen (1956). (**B**) The speed of light measurement at the National Physical Laboratory (NPL) in the United Kingdom (1979). This experiment and a similar one at the National Bureau of Standards (now NIST) in the United States established the value used in the speed-of-light definition of the meter. (**C**) A modern Josephson volt apparatus. (**D**) The NPL watt balance apparatus.

The only measurement required is the length of the rods making up the capacitor.

The challenge, as with many experiments of this type, is the care needed in construction, measurement, and alignment of a system to the highest possible accuracy. A number of standards laboratories around the world have done experiments of this type and have achieved uncertainties of a few parts in 10^8. Some such measurements are still in progress.

An absolute volt realization was made by Clothier (*17*) at the National Measurement Laboratory, Australia, with an electrometer in which one of the electrodes is a pool of mercury. A vertical electric field, U, is applied, and the change in height of the mercury, d, is measured. The rise of the mercury surface is given by

$$|U| = \sqrt{\frac{2\rho g}{\varepsilon_0 \varepsilon_r}}\, d\, s^{1/2} \qquad (6)$$

where ρ is the density of the mercury, g the acceleration due to gravity, s the interelectrode spacing, and ε_r the relative permittivity of the gas between the electrode and the mercury pool. In practice, the difference between voltages is measured to reduce systematic effects. After considerable effort, this experiment gave a result with an uncertainty of a few parts in 10^7. A capacitance balance measurement at the Physikalisch-Technische Bundesanstalt, Germany (*18*), has achieved a similar accuracy and is in good agreement. It seems unlikely that there will be much progress unless a new method is invented.

Quantum voltage and resistance standards. The start of electrical quantum metrology began with the Josephson effect; indeed, this helped to introduce quantum metrology as a concept. In 1962, Josephson predicted (*19*) that in the presence of an applied microwave field, a direct superconducting tunneling current could pass between superconductors separated by an insulating barrier. This current can only pass when the voltage V across the barrier satisfies the relationship

$$2eV = nh\nu \qquad (7)$$

where e is the electron charge, h the Planck constant, ν the applied frequency, and n an integer. It was recognized that voltage standards could be based on this effect (Fig. 1C). These soon showed that the standard cells used by national laboratories to maintain the volt were drifting at a greater rate than had been believed. A number of experiments found no corrections to expression 7 or dependence on material or experimental conditions at a level of up to parts in 10^{16}. In 1972, a number of countries used the Josephson effect to maintain the volt and agreed on an assigned value for $2e/h$ so that their voltages were in agreement. They are not necessarily the correct SI value; hence, the agreed-upon value is referred to as a "representation" of the volt. Not all countries adopted the same value, but in 1990 international agreement was reached, and the defined value adopted for the frequency-to-voltage conversion $K_{J\text{-}90}$, based on the best knowledge of the volt from absolute volt experiments, was

$$K_{J-90} = 483597.9 \text{ GHz V}^{-1} \qquad (8)$$

This assigned value can be compared with the current best estimate of the value in SI volts; the 2002 Committee on Data for Science and Technology (CODATA) evaluation (*20*) gives

$$K_J = \frac{2e}{h} = K_{J-90}\left[1 - \left((4.3 \pm 8.5) \times 10^{-8}\right)\right] \qquad (9)$$

so there is no evidence that the 1990 value is significantly in error.

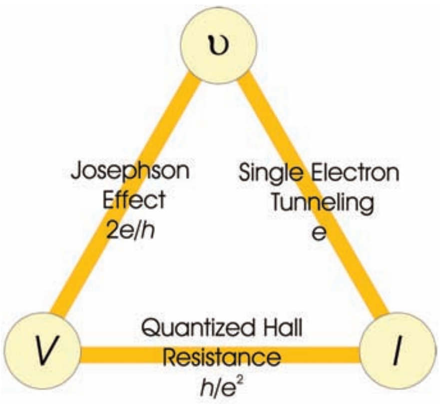

Fig. 2. The metrological triangle. The points of the triangle represent the electrical variables, and the sides represent the experiments embodying quantum effects that link them.

A second electrical quantum effect was demonstrated by von Klitzing in 1980 (*21*), establishing a quantum-based resistance. The quantum Hall effect produces resistance steps in the current-voltage characteristic of a two-dimensional electron gas semiconductor device with values given by

$$R_H = \frac{h}{ie^2} \qquad (10)$$

where h is the Planck constant, e the electron charge, and i an integer index through the steps. Experimental work showed that the resistance of these devices was material and condition-independent to better than one part in 10^9 and, perhaps because of the success of the Josephson effect volt, the quantum Hall resistance standard was quickly adopted, even though the theory behind Eq. 10 is less well established (*22*, *23*). A conventional value of the von Klitzing constant R_{K-90} was adopted in 1990 as a representation of the ohm at the same time as the Josephson constant for the volt (*24*). The conventional value is given by

$$R_{K-90} = 25812.807\ \Omega \qquad (11)$$

Again, this value is based on absolute realizations of the ohm, such as the calculable capacitor, and the CODATA 2002 evaluation gives for the von Klitzing constant

$$\begin{aligned} R_K &= \frac{h}{e^2} \\ &= R_{K-90}\left[1 + \left((1.74 \pm 0.33) \times 10^{-8}\right)\right] \end{aligned} \qquad (12)$$

Fortunately, the small difference between the assigned value and the SI value is inconsequential for most purposes and is well within the uncertainty assigned in 1990.

A further connection enabled by the quantum Hall effect is the link to the fine-structure constant α given by

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha} \qquad (13)$$

This means that experiments from other parts of physics that determine the fine-structure constant give information on the ohm. The value of the fine-structure constant obtained from the measurement of the anomalous magnetic moment of the electron (*25*) is accurate to 3.8×10^{-9}, whereas the mean of the R_K data gives an uncertainty on the fine-structure constant of 1.8×10^{-8}.

Quantum current standards and the "metrological triangle." The exactness of the expressions $K_J = 2e/h$ and $R_K = h/e^2$ cannot be ensured by theory, so a long-standing goal of electrical metrology is to test the consistency of those relationships using the "metrological triangle" (Fig. 2). The experiment is to compare the voltage produced by a quantized current (controlled passage of counted electrons) passing through a quantum Hall effect resistor with a Josephson voltage. To do this at a meaningful level will require a quantum standard of current accurate at about one part in 10^8, a goal that is actively being pursued in a number of laboratories (*26*). An experiment that demonstrated the principle of electron counting was the charging of a capacitor with electrons counted with a single-electron transistor (*27*), although the rates necessary for closure of the metrological triangle have not been achieved. The present state of the art is about an order of magnitude short of the parts in 10^8 accuracy needed. Electron counting gives a current I comprising electrons in step with a driving frequency ν

$$I = \nu e \qquad (14)$$

This expression combined with expressions 7 and 10 gives a redundancy that enables the exactness of the three relationships to be tested. Although there could still be an offset in all three expressions coming from some as-yet-unknown physics, it would be very surprising if independent systematic errors from three experiments could cancel each other out.

The Kilogram: The Remaining Artifact Base Unit

The kilogram is the remaining artifact standard of the SI (Fig. 3), as is clear from its definition: "The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram" [3rd CGPM, 1901 (*28*)].

By definition, the kilogram standard cannot change mass; it is always one kilogram in SI units. However, by considering the physics of the situation, we expect that there is a sense in which its mass could be drifting. Perhaps the intermittent cleaning process to which it is subjected removes some material from the kilogram artifact, or perhaps it absorbs some impurities from the atmosphere. We would consider these mass changes, but this can only be tested experimentally by having some unit of mass that we consider as more stable, for example, a mass based on atomic masses or on the mass equivalent of energy.

Fig. 3. The prototype kilogram. The only object that has a mass of exactly one kilogram. The prototype kilogram, the last artifact unit of the SI, is kept in a vault at the Bureau International des Poids et Mesures in Sèvres near Paris.

Link with electrical unit: the watt balance. Early realizations of the ampere were based on balancing the forces between accurately made coils. In the 1960s, nuclear magnetic resonance (NMR) methods of flux density measurement were introduced, and in 1969 a recommended value of the gyromagnetic ratio of the proton was agreed upon to allow the ampere to be maintained in terms of the flux density produced by a coil. This method was used in standards laboratories to monitor standard cells, which were the practical electrical unit. Until 1990, the unit of flux density (which is volt second per meter squared) disseminated directly by NMR was in closer agreement with the SI than the volt itself could be disseminated via standard cells.

A new method of realizing the watt was proposed by Kibble in 1975 (*29*, *30*), building on the techniques developed for the earlier current measurements with NMR. The method, known as the watt balance, in effect links the mechanical and electrical units of power. Following the discovery of the quantum Hall effect, this can be reinterpreted as a linking of the kilogram to the Planck constant h. In this experiment, a coil is suspended from the arm of a balance in a magnetic field of flux density B, and the current I required to produce a force to balance a mass m is measured. Subsequently, the coil is moved at a measured velocity through the field, and the voltage E generated is measured. These two operations give, for the weighing mode

$$mg = Bk_{\mathrm{c}}I \qquad (15)$$

where k_{c} is the coil constant, defining its geometrical properties, and g is the acceleration due to gravity. For the moving mode

$$E = Bk_{\mathrm{c}}\frac{\mathrm{d}z}{\mathrm{d}t} \qquad (16)$$

The elegance of this method is that the geometry factor and the magnetic field can be canceled out, removing the requirement to measure the coil constant that limited ampere balances. Combining Eqs. 15 and 16 gives

$$EI = mg\frac{\mathrm{d}z}{\mathrm{d}t} \qquad (17)$$

Measuring the mechanical quantities m, g, and $\mathrm{d}z/\mathrm{d}t$ thus gives an absolute measurement of electrical power EI.

If the voltage is measured in terms of the Josephson volt and the current in terms of the Josephson volt and quantum Hall resistance, the result determines $K_{\mathrm{J}}^2R_{\mathrm{K}}$, which by substitution of the definitions above gives $4/h$. Thus, the left-hand side of Eq. 17 can be considered fixed to the Planck constant, and measuring g and $\mathrm{d}z/\mathrm{d}t$ gives an electrical method of measuring the kilogram.

Again there are formidable practical difficulties, especially in ensuring that the product Bk_{c} is the same in both cases. First-generation measurements have been made by the National Physical Laboratory, UK (Fig. 1D), and the National Institute of Standards and Technology (NIST), USA, giving a combined uncertainty of 8.7 × 10^{-8}. A number of laboratories are now undertaking measurements of this type; these have been reviewed by Eichenberger (*31*). Achieving the target accuracy of one part in 10^8 for the next generation of measurements would allow monitoring of the stability of the prototype kilogram at a significant level compared with the uncertainties associated with its maintenance.

Counting atoms: the Avogadro number method. An alternative to electrical methods for producing a nonartifact mass standard is provided by methods based on counting atomic masses. The most developed of these is the crystallographic method, which is being pursued by an international collaboration (*32*). This method relies on measurements of the lattice spacing of an ultrapure silicon crystal sphere of remarkable roundness. The radius r of the sphere is measured by optical interferometry, and the lattice spacing a determined by x-ray interferometry; hence, the mass m of the crystal will be known in terms of the molar mass of silicon M(Si) and the Avogadro constant N_{A}, that is

$$m = \frac{nM(\mathrm{Si})\,\frac{4}{3}\pi r^3}{N_{\mathrm{A}}a^3} \qquad (18)$$

where n is the number of atoms per unit cell.

The lattice spacing is not a fundamental constant, as it requires standard conditions of pressure and temperature. Considerable effort is needed to determine the measured components in Eq. 18 to the required level of accuracy. Current effort is focused on repeating the experiment, but with a piece of monoisotopic ^{28}Si, because the determination of the isotope ratio is currently one of the principal limiting factors. The silicon sphere will be artifact-like in that it will be difficult to ensure that it remains unchanged and to compare it with secondary standards. Although it could be reproduced if damaged, this would require major and time-consuming effort.

Surprisingly, perhaps, the link between the Avogadro and Planck constants is provided by the Rydberg constant, a constant of atomic spectroscopy given by

$$R_\infty = \frac{m_{\mathrm{e}}c\alpha^2}{2h} \qquad (19)$$

This expression shows the link between the Planck constant and the electron mass m_{e}. Because atomic mass ratios and the electron-to-proton mass ratios are well known, the electron mass provides a link to the Avogadro. So the Planck constant may be written

$$h = \frac{cA_{\mathrm{r}}(\mathrm{e})M_{\mathrm{u}}\alpha^2}{2R_\infty N_{\mathrm{A}}} \qquad (20)$$

where A_{r}(e) is the relative atomic mass of the electron and M_{u} is the molar mass constant, 10^{-3} kg mol^{-1} exactly.

Another less-developed method of this type is also under investigation as a replacement mass standard, an ion accumulation method (*33*). In this method, the current in an ion beam is accurately measured and the accumulated material is then weighed. An element with only one natural isotope can be used to prevent the isotope ratio problems that have been encountered in the Avogadro method. However, this method is presently limited by the small mass collected and by the difficulty of ensuring that all the ions in the measured current are collected.

A redefinition of the kilogram could be considered based on a fixed value of either the Planck constant or the Avogadro number. In practice, it is not necessary to use the defined method for realization. If, for example, the kilogram were defined by fixing the mass of the carbon-12 atom, which would provide a nice intuitive definition, then in practice the watt balance could still be used to realize the kilogram. Relationships between the fundamental constants, such as Eq. 20, then allow conversions between realization and definition, as well as consistency checks with experiments based on different physical effects.

A speculative pressure method. Future quantum standards based on new or revisited quantum effects will likely take us by surprise, as did some of the effects presently used. One highly speculative possibility that has been considered is an absolute pressure standard based on superfluid helium. Given a measurable area, a pressure standard also provides a force and hence a mass standard.

The Josephson effect in a superconductor that provides the basis of the voltage standards described above has an equivalent in superfluid helium. In this case, the oscillation is in mass flow through a nanoaperture in response to the chemical potential across it. This was proposed as a quantum standard in the 1960s, but the effect could not be observed because of the difficulty of manufacturing apertures small enough to provide the weak link. Recently, the effect has been observed in helium-3 (*34*, *35*) and helium-4 (*36*), with the weak link provided by membranes with arrays of apertures, each on the order of 100 nm in diameter. The Josephson frequency ω_J is given by

$$\omega_J \equiv \frac{\Delta\mu}{\hbar} = \frac{m\Delta P}{\rho\hbar} \tag{21}$$

where ΔP is the pressure difference across the membrane, m the particle mass, and ρ the density of the liquid. In the same form as Eq. 7, this gives

$$\frac{m}{\rho}P = nh\nu \tag{22}$$

To provide a pressure standard, the density of the liquid would have to be measured to high accuracy. The effect is being developed as a candidate rotation sensor, but I am not aware of any standards-focused work that is being undertaken.

Temperature in Terms of Fundamental Constants

Temperature and optical quantities have associated fundamental constants, but the technology that may provide the link allowing the units to be defined in terms of atomic or quantum effects is much further from being established.

Although the kilogram is the last artifact base unit in the SI, the kelvin definition does not meet the criterion of a quantum standard in that it is material dependent and so could be considered "artifact-like." The kelvin is defined as "The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water" [13th CGPM, 1967 (*10*)].

The realization of this definition is limited ultimately in practice by the chemical purity and isotopic content of the water, so that measurable differences exist between triple-point cells from different sources. However, unlike an artifact, a new triple-point cell can be made without reference to another.

To realize the kelvin, one must establish a triple-point equilibrium, but to measure any other temperature it is also necessary to have a primary thermometer to compare the unknown temperature with the temperature of the triple point. A primary thermometer is one whose temperature can be related to physical parameters without unknown constants being introduced; these, therefore, have the potential to link temperature to fundamental constants. Methods of thermodynamic thermometry have been reviewed by Rusby *et al.* (*37*). The most common primary thermometer is the gas thermometer in which the temperature T is related to the pressure p and volume V of a gas of amount of substance n by

$$p = RT\frac{n}{V} \times \left[1 + \frac{n}{V}B(T) + \frac{n^2}{V^2}C(T) + \cdots\right] \tag{23}$$

where R is the gas constant, and $\mathrm{B}(T)$, $\mathrm{C}(T)$, and so on are virial coefficients that are calculable for a real gas. More precise measurement of the gas constant comes from acoustic thermometry, which measures the speed of sound c and relates this to temperature via the equation of state. In the low pressure limit, this is given by

$$c^2 = \frac{\gamma RT}{M} \tag{24}$$

where γ is the ratio of specific heats and M is the molar mass of the gas but, in practice, similarly to the gas thermometer case, a series of acoustic virial coefficients must be used for a real gas.

A primary thermometer based on a different principle is the total radiation thermometer. Here, the temperature is related to the total radiation of a black body, M, by

$$M = \sigma T^4, \quad \sigma = \frac{\pi^2 k^4}{60\hbar^3 c^2} \tag{25}$$

where σ is the Stefan-Boltzmann constant. In this method, the radiation heats a black body absorber, and the heating effect is measured by comparison of the temperature rise it produces to that produced by electrical heating (*38*). This method also provides a link to optical quantities, for if the black body is replaced by a monochromatic light source, then the optical power is measured in terms of electrical power.

Another primary thermometer that has been investigated for metrological use is the noise thermometer, where the mean square electrical noise voltage $\overline{V^2}$ across a resistance R is given by

$$\overline{V^2} = 4kTR\Delta f \tag{26}$$

where Δf is the measurement bandwidth and k the Boltzmann constant. A recent proposition is to use tunnel junction shot noise as a primary thermometer. (*39*).

In all these cases, a fundamental constant relates the measured parameters to temperature; these are conversion factors between thermal and other units. Given the definition of the kelvin as it stands, it is not possible to measure these constants to a greater accuracy than that to which the triple point can be established. If one of these methods were advanced to a stage where it was able to give a resolution better than the stability of the triple-point cell, then it would be appropriate to redefine the kelvin in terms of one of the constants R or k (which are related by the relationship $R = N_A k$). The appropriate primary thermometer would then realize the kelvin and could be used to calibrate fixed points on which a temperature scale would be based. Most likely, a system of fixed points will still be used, as the primary thermometer will not be operable over a wide range of temperatures and is likely to be time consuming and expensive to realize.

It is the unusual properties of the quantity temperature that limit the use of an atomic or quantum definition. The concept of temperature is only applicable to an ensemble. However, under certain conditions, the time series properties of a single state are the same as the properties of an ensemble, and this may provide a route to a quantized method of temperature measurement.

Conclusions

This short review has summarized thousands of person-years of work linking measurement to atomic and quantum phenomena. As with experiments at the limit of precision, meticulous care is required, and the search for systematic effects seems endless. However, it seems likely that within decades we will have an internationally agreed-upon unit system based entirely on atomic and quantum phenomena, as was envisaged more than a century ago. This will provide a system with a stability and internal consistency based on fundamental constants and, thus, able to provide standards at the low levels of uncertainty required for scientific and technological progress.

Although the route to an SI based on fundamental constants seems nearly complete with the replacement of the artifact kilogram on the horizon, metrology and fundamental constant determinations are far from their ultimate limits.

Both the absolute temperature measurements and the watt balance are unlikely to show the orders-of-magnitude improvement that has been seen with the microwave cesium frequency standard and quantum electrical standards. They are both apparently close to their practical limit already, so any changes to the definitions of the units should be made with caution, as they may not be long-lasting. We await new methods and effects to bring the advances that have been seen in frequency and electrical metrology to other areas of measurement.

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REVIEW

Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

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Quantum mechanics, through the Heisenberg uncertainty principle, imposes limits on the precision of measurement. Conventional measurement techniques typically fail to reach these limits. Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using quantum strategies that employ "quantum tricks" such as squeezing and entanglement.

Measurement is a physical process, and the accuracy to which measurements can be performed is governed by the laws of physics. In particular, the behavior of systems at small scales is governed by the laws of quantum mechanics, which place limits on the accuracy to which measurements can be performed. These limits to accuracy take two forms. First, the Heisenberg uncertainty relation (*1*) imposes an intrinsic uncertainty on the values of measurement results of complementary observables such as position and momentum, or the different components of the angular momentum of a rotating object (Fig. 1). Second, every measurement apparatus is itself a quantum system: As a result, the uncertainty relations together with other quantum constraints on the speed of evolution [such as the Margolus-Levitin theorem (*2*)] impose limits on how accurately we can measure quantities, given the amount of physical resources, such as energy, at hand to perform the measurement.

One important consequence of the physical nature of measurement is the so-called quantum back action: The extraction of information from a system can give rise to a feedback effect in which the system configuration after the measurement is determined by the measurement outcome. For example, the most extreme case (the so-called von Neumann or projective measurement) produces a complete determination of the post-measurement state. When performing successive measurements, quantum back action can be detrimental, because earlier measurements can negatively influence successive ones. A common strategy to get around the negative effect of back action

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