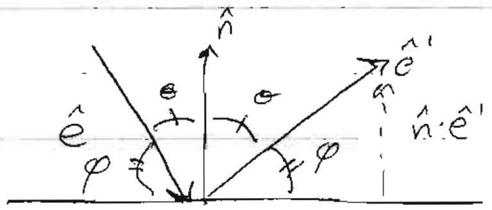
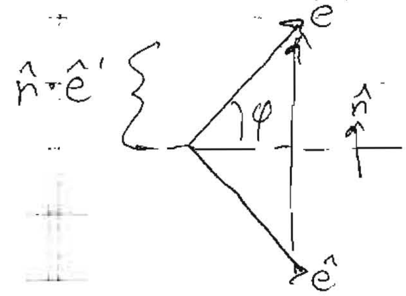
5.13)



$$\hat{e}'\cdot\hat{n} = \cos\theta$$

$$\hat{e}\cdot\hat{n} = \cos(180 - \theta) = -\cos\theta$$

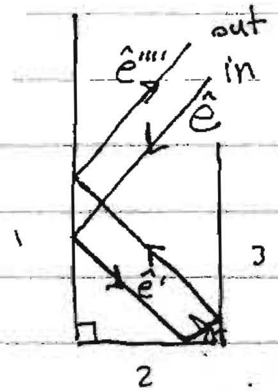


$$\hat{e}' - \hat{e} = 2|(\hat{n}\cdot\hat{e})|\,\hat{n}\,(-1)$$

$$\boxed{\begin{aligned} \hat{e}' &= \hat{e} + 2(\hat{n}\cdot\hat{e})\,\hat{n} \\ \text{or}\quad \hat{e}' &= \hat{e} - 2|\hat{n}\cdot\hat{e}|\,\hat{n} \end{aligned}}$$

note $\hat{n}, \hat{e}, \hat{e}'$ all have same <u>length</u>
they just differ in angle.

5.14)



from 5.13

show $\hat{e}\cdot\hat{e}'''' = -1$

$\hat{e}' = \hat{e} - 2(\hat{n}\cdot\hat{e})\hat{n}$

1. $\hat{e}' = \hat{e} - 2(\hat{n}_1\cdot\hat{e})\hat{n}_1$
2. $\hat{e}'' = \hat{e}' - 2(\hat{n}_2\cdot\hat{e}')\hat{n}_2$
3. $\hat{e}''' = \hat{e}'' - 2(\hat{n}_3\cdot\hat{e}'')\hat{n}_3$
4. $\hat{e}'''' = \hat{e}''' - 2(\hat{n}_4\cdot\hat{e}''')\hat{n}_4$

$\hat{n}_4 = \hat{n}_1$

$\hat{n}_i\cdot\hat{n}_j = \delta_{ij}$ (at 90°)

3 → 4 $\hat{e}'''' = e'' - 2\hat{n}_3\cdot\hat{e}''\,\hat{n}_3 - 2\hat{n}_4\,\hat{n}_4\cdot\hat{e}''$
$+4\,\cancel{\hat{n}_4\cdot\hat{n}_3}^{\,0}\,\hat{n}_3\cdot\hat{e}''\,\hat{n}_4$

and so on $\hat{e}'''' = \hat{e}' - 2(\hat{n}_2\cdot\hat{e}')\hat{n}_2 - 2(\hat{n}_3\cdot\hat{e}')\hat{n}_3$
$-2\hat{n}_4\,\hat{n}_4\cdot\hat{e}'$

$\hat{e}'''' = \hat{e} - \cancel{2\hat{n}_1\cdot\hat{e}\,\hat{n}_1} - 2\hat{n}_2\cdot\hat{e}\,\hat{n}_2 - 2\hat{n}_3\cdot\hat{e}\,\hat{n}_3$
$-\cancel{2\hat{n}_1\cdot\hat{e}\,\hat{n}_1} + \cancel{4\hat{n}_1\cdot\hat{e}\,\hat{n}_1}$

$\hat{e}''''\cdot\hat{e} = 1 - 2\left[\underbrace{(\hat{n}_2\cdot e)^2}_{\cos^2\theta_1} + \underbrace{(\hat{n}_3\cdot e)^2}_{\cos^2\theta_2}\right]$

$\theta_2 = 90 \pm \theta_1$ if $\hat{n}_3\cdot\hat{n}_2 = 0$

$\Rightarrow 1 - 2[\cos^2\theta_1 + \sin^2\theta_1] = -1$

$\Rightarrow \boxed{\hat{e}''''\cdot\hat{e} = -1}$