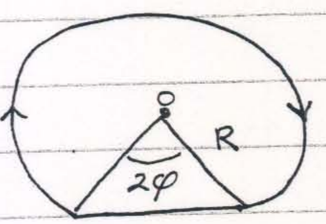
Sols PQ (6) 4BL '07 MHM

3.222) $I = 5.0\,A \quad R = .12\,m \quad 2\varphi = 90^\circ$



I find magnetic field at O

$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{I}{r^3}\, d\bar{l} \times \bar{r}$$

our integral is by superposition

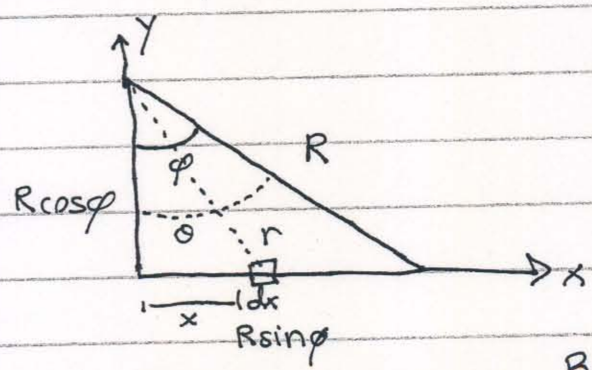
$$\oint_{circle} - \int_{circle\,-\varphi}^{\varphi} + \int_{line\,-\varphi}^{\varphi} = B \text{ at } O$$

for circle $d\bar{l} \times \bar{r} = \begin{vmatrix} \hat{r} & \hat{\varphi} & \hat{z} \\ dr & r\,d\varphi & dz \\ r & 0 & 0 \end{vmatrix} = -r^2 d\varphi\, \hat{z} \Big|_{r=R}$

$$B_{circle} = \frac{\mu_0}{4\pi}\frac{I}{R}\oint d\varphi\,(-\hat{z})$$

$$B_{circle \text{ from } \varphi \text{ to } -\varphi} = \frac{\mu_0}{4\pi}\frac{I}{R}\int_{-\varphi}^{\varphi} d\varphi \; -\hat{z}$$

$$\oint - \int_{-\varphi}^{\varphi} = \frac{\mu_0 I}{2\pi R}(\pi - \varphi)$$



$d\bar{x} \times \bar{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx & 0 & 0 \\ x & R\cos\varphi & 0 \end{vmatrix} = -R\cos\varphi\, dx$

$$r = \sqrt{x^2 + R^2\cos^2\varphi}$$

$$B_{line} = \int_{-\varphi}^{\varphi} \frac{\mu_0 I}{4\pi} \frac{(-R\cos\varphi\, dx)\hat{z}}{(x^2 + R^2\cos^2\varphi)^{3/2}}$$

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$$B_{line} = \int_{-\varphi}^{\varphi} \frac{-\mu_0 I R\cos\varphi}{4\pi}\hat{z} \frac{dx}{(x^2 + R^2\cos^2\varphi)^{3/2}}$$

substitution $x = R\cos\varphi \tan\sigma$ $\quad \varphi \rightarrow \frac{R\sin\varphi}{R\cos\varphi} = \tan\sigma_{upper}$

$dx = R\cos\varphi \sec^2\sigma\, d\sigma$ $\quad -\varphi \rightarrow -\frac{R\sin\varphi}{R\cos\varphi} = \tan\sigma_{lower}$

limits remain unchanged... weird!

$$B_{line} = \frac{-\mu_0 I R\cos\varphi\,\hat{z}}{4\pi} \frac{R\cos\varphi}{(R\cos\varphi)^3} \int_{-\varphi}^{\varphi} \frac{\sec^2\sigma\, d\sigma}{\sec^3\sigma}$$

remember! $\int \sin\theta\, d\theta = -\cos\theta$ $\quad \int \cos\theta\, d\theta = \sin\theta$

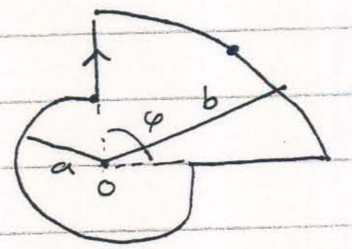
$$B_{line} = -\frac{\mu_0 I}{2\pi R}\hat{z} \tan\varphi$$

$$\boxed{B_{total} = -\frac{\mu_0 I}{2\pi R}\hat{z}\left(\pi - \varphi + \tan\varphi\right)}$$

$$\boxed{B_{total} = -28\mu T\hat{z}}$$

$\mu_0 \cong 1.26 \times 10^{-6} \frac{m\,kg}{A^2 s^2}$

3.223) a)

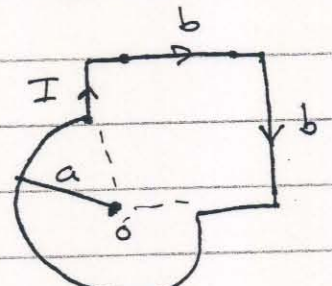


$$B_{at\,o} = \oint_a - \int_a^{\varphi} + \int_b^{\varphi}$$ see previous problem for how to do integrals

$$= (-\hat{z})\left[\frac{\mu_0 I}{2\pi a}(\pi - \varphi/2) + \frac{\mu_0 I}{2\pi b}\,\varphi/2\right]$$

$$\boxed{B_{at\,o} = -\frac{\mu_0 I}{4\pi}\left[\frac{2\pi-\varphi}{a} + \frac{\varphi}{b}\right]\hat{z}}$$

b)



$$B_{at\,o} = \oint_a - \int_a^{90^\circ} + 2\int_{line}^{\pi/4}$$ ← 45° see integrals from 3.222

$$= -\frac{\mu_0 I}{4\pi}\hat{z}\left(\left[\frac{2\pi - \pi/2}{a}\right] + \frac{\sqrt{2}}{b}\right)$$

$$\boxed{B_{at\,o} = -\frac{\mu_0 I}{4\pi}\hat{z}\left(\frac{3\pi}{2a} + \frac{\sqrt{2}}{b}\right)}$$