PQ(8) 4BL MHM '07

5.4. Determine the luminosity of a surface whose luminance depends on direction as $L = L_0 \cos \theta$, where θ is the angle between the radiation direction and the normal to the surface.

5.5. A certain luminous surface obeys Lambert's law. Its luminance is equal to L. Find:

(a) the luminous flux emitted by an element ΔS of this surface into a cone whose axis is normal to the given element and whose aperture angle is equal to θ;

(b) the luminosity of such a source.

5.6. An illuminant shaped as a plane horizontal disc $S = 100$ cm^2 in area is suspended over the centre of a round table of radius $R = 1.0$ m. Its luminance does not depend on direction and is equal to $L = 1.6 \cdot 10^4$ cd/m^2. At what height over the table should the illuminant be suspended to provide maximum illuminance at the circumference of the table? How great will that illuminance be? The illuminant is assumed to be a point source.

5.7. A point source is suspended at a height $h = 1.0$ m over the centre of a round table of radius $R = 1.0$ m. The luminous intensity I of the source depends on direction so that illuminance at all points of the table is the same. Find the function $I(\theta)$, where θ is the angle between the radiation direction and the vertical, as well as the luminous flux reaching the table if $I(0) = I_0 = 100$ cd.

5.8. A vertical shaft of light from a projector forms a light spot $S = 100$ cm^2 in area on the ceiling of a round room of radius $R = 2.0$ m. The illuminance of the spot is equal to $E = 1000$ lx. The reflection coefficient of the ceiling is equal to $\rho = 0.80$. Find the maximum illuminance of the wall produced by the light reflected from the ceiling. The reflection is assumed to obey Lambert's law.

5.9. A luminous dome shaped as a hemisphere rests on a horizontal plane. Its luminosity is uniform. Determine the illuminance at the centre of that plane if its luminance equals L and is independent of direction.

5.10. A Lambert source has the form of an infinite plane. Its luminance is equal to L. Find the illuminance of an area element oriented parallel to the given source.

5.11. An illuminant shaped as a plane horizontal disc of radius $R = 25$ cm is suspended over a table at a height $h = 75$ cm. The illuminance of the table below the centre of the illuminant is equal to $E_0 = 70$ lx. Assuming the source to obey Lambert's law, find its luminosity.

5.12. A small lamp having the form of a uniformly luminous sphere of radius $R = 6.0$ cm is suspended at a height $h = 3.0$ m above the floor. The luminance of the lamp is equal to $L = 2.0 \cdot 10^4$ cd/m^2 and is independent of direction. Find the illuminance of the floor directly below the lamp.

5.13. Write the law of reflection of a light beam from a mirror in vector form, using the directing unit vectors $\mathbf{e}$ and $\mathbf{e}'$ of the incident and reflected beams and the unit vector $\mathbf{n}$ of the outside normal to the mirror surface.

5.14. Demonstrate that a light beam reflected from three mutually perpendicular plane mirrors in succession reverses its direction.

using result from 5.13

5.15. At what value of the angle of incident θ_1 is a shaft of light reflected from the surface of water perpendicular to the refracted shaft?

5.16. Two optical media have a plane boundary between them. Suppose θ_{1cr} is the critical angle of incidence of a beam and θ_1 is the angle of incidence at which the refracted beam is perpendicular to the reflected one (the beam is assumed to come from an optically denser medium). Find the relative refractive index of these media if $\sin \theta_{1cr} / \sin \theta_1 = \eta = 1.28$.

5.17. A light beam falls upon a plane-parallel glass plate $d = 6.0$ cm in thickness. The angle of incidence is $\theta = 60°$. Find the value of deflection of the beam which passed through that plate.

5.18. A man standing on the edge of a swimming pool looks at a stone lying on the bottom. The depth of the swimming pool is equal to h. At what distance from the surface of water is the image of the stone formed if the line of vision makes an angle θ with the normal to the surface?

5.19. Demonstrate that in a prism with small refracting angle θ the shaft of light deviates through the angle $\alpha \simeq (n - 1)\theta$ regardless of the angle of incidence, provided that the latter is also small.

5.20. A shaft of light passes through a prism with refracting angle θ and refractive index n. Let α be the diffraction angle of the shaft. Demonstrate that if the shaft of light passes through the prism symmetrically,

(a) the angle α is the least;

(b) the relationship between the angles α and θ is defined by Eq. (5.1e).

5.21. The least deflection angle of a certain glass prism is equal to its refracting angle. Find the latter.

5.22. Find the minimum and maximum deflection angles for a light ray passing through a glass prism with refracting angle $\theta = 60°$.

5.23. A trihedral prism with refracting angle 60° provides the least deflection angle 37° in air. Find the least deflection angle of that prism in water.

5.24. A light ray composed of two monochromatic components passes through a trihedral prism with refracting angle $\theta = 60°$. Find the angle $\Delta\alpha$ between the components of the ray after its passage through the prism if their respective indices of refraction are equal to 1.515 and 1.520. The prism is oriented to provide the least deflection angle.

5.25. Using Fermat's principle derive the laws of deflection and refraction of light on the plane interface between two media.