PQ (7) 4BL '07 MHM

4.149. A wooden core (Fig. 4.36) supports two coils: coil *1* with inductance L_1 and short-circuited coil *2* with active resistance R and inductance L_2. The mutual inductance of the coils depends on

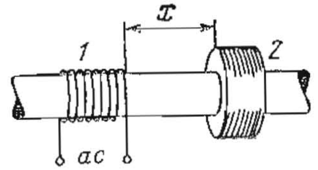

Fig. 4.36.

$$\int_{z_1}^{z_2} \frac{dz}{\sqrt{z}} = 2(\sqrt{z_2}-\sqrt{z_1})$$

the distance x between them according to the law $L_{12}(x)$. Find the mean (averaged over time) value of the interaction force between the coils when coil *1* carries an alternating current $I_1 = I_0 \cos \omega t$.

4.3. ELASTIC WAVES. ACOUSTICS

- Equations of plane and spherical waves:

$$\xi = a \cos(\omega t - kx), \qquad \xi = \frac{a_0}{r} \cos(\omega t - kr). \tag{4.3a}$$

In the case of a homogeneous absorbing medium the factors $e^{-\gamma x}$ and $e^{-\gamma r}$ respectively appear in the formulas, where γ is the wave damping coefficient.

- Wave equation:

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}. \tag{4.3b}$$

- Phase velocity of longitudinal waves in an elastic medium ($v_{\parallel}$) and transverse waves in a string ($v_{\perp}$):

$$v_{\parallel} = \sqrt{E/\rho}, \quad v_{\perp} = \sqrt{T/\rho_1}, \tag{4.3c}$$

where E is Young's modulus, ρ is the density of the medium, T is the tension of the string, ρ_1 is its linear density.

- Volume density of energy of an elastic wave:

$$w = \rho a^2 \omega^2 \sin^2(\omega t - kx), \quad \langle w \rangle = {}^1/_2 \rho a^2 \omega^2. \tag{4.3d}$$

- Energy flow density, or the Umov vector for a travelling wave:

$$\mathbf{j} = w\mathbf{v}, \quad \langle \mathbf{j} \rangle = {}^1/_2 \rho a^2 \omega^2 \mathbf{v}. \tag{4.3e}$$

- Standing wave equation:

$$\xi = a \cos kx \cdot \cos \omega t. \tag{4.3f}$$

- Acoustical Doppler effect:

$$\nu = \nu_0 \frac{v + v_{ob}}{v - v_s}. \tag{4.3g}$$

- Loudness level (in bels):

$$L = \log(I/I_0). \tag{4.3h}$$

- Relation between the intensity I of a sound wave and the pressure oscillation amplitude $(\Delta p)_m$:

$$I = (\Delta p)_m^2 / 2\rho v. \tag{4.3i}$$

4.150. How long will it take sound waves to travel the distance l between the points A and B if the air temperature between them varies linearly from T_1 to T_2? The velocity of sound propagation in air is equal to $v = \alpha\sqrt{T}$, where α is a constant.

4.151. A plane harmonic wave with frequency ω propagates at a velocity v in a direction forming angles α, β, γ with the x, y, z axes. Find the phase difference between the oscillations at the points of medium with coordinates x_1, y_1, z_1 and x_2, y_2, z_2.

4.152. A plane wave of frequency ω propagates so that a certain phase of oscillation moves along the x, y, z axes with velocities v_1, v_2, v_3 respectively. Find the wave vector $\mathbf{k}$, assuming the unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ of the coordinate axes to be assigned.

4.153. A plane elastic wave $\xi = a \cos(\omega t - kx)$ propagates in a medium K. Find the equation of this wave in a reference frame K' moving in the positive direction of the x axis with a constant velocity V relative to the medium K. Investigate the expression obtained.

4.154. Demonstrate that any differentiable function $f(t + \alpha x)$, where α is a constant, provides a solution of wave equation. What is the physical meaning of the constant α?

4.155. The equation of a travelling plane sound wave has the form $\xi = 60 \cos(1800t - 5.3x)$, where ξ is expressed in micrometres, t in seconds, and x in metres. Find:

(a) the ratio of the displacement amplitude, with which the particles of medium oscillate, to the wavelength;

(b) the velocity oscillation amplitude of particles of the medium and its ratio to the wave propagation velocity;

(c) the oscillation amplitude of relative deformation of the medium and its relation to the velocity oscillation amplitude of particles of the medium.

4.156. A plane wave $\xi = a \cos(\omega t - kx)$ propagates in a homogeneous elastic medium. For the moment $t = 0$ draw

(a) the plots of ξ, $\partial\xi/\partial t$, and $\partial\xi/\partial x$ vs x;

(b) the velocity direction of the particles of the medium at the points where $\xi = 0$, for the cases of longitudinal and transverse waves;

(c) the approximate plot of density distribution $\rho(x)$ of the medium for the case of longitudinal waves.

4.157. A plane elastic wave $\xi = ae^{-\gamma x} \cos(\omega t - kx)$, where a, γ, ω, and k are constants, propagates in a homogeneous medium. Find the phase difference between the oscillations at the points where the particles' displacement amplitudes differ by $\eta = 1.0\%$, if $\gamma = 0.42\ \text{m}^{-1}$ and the wavelength is $\lambda = 50$ cm.

4.158. Find the radius vector defining the position of a point source of spherical waves if that source is known to be located on the straight line between the points with radius vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ at which the oscillation amplitudes of particles of the medium are equal to a_1 and a_2. The damping of the wave is negligible, the medium is homogeneous.