Dis 6A 3.1

1.27) $y = ax - bx^2$

$w = -g\hat{y} = \frac{\partial^2 x}{\partial t^2}\hat{x} + \frac{\partial^2 y}{\partial t^2}\hat{y}$ $\quad g = \text{constant}$

find $v$ at origin $(x=0, y=0)$

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$\frac{\partial^2 x}{\partial t^2} = 0 \qquad \frac{\partial^2 y}{\partial t^2} = -g$

$x = x_0 + v_x t$ $\quad v_x = \text{constant}$

$y = y_0 + v_y t - \frac{1}{2} g t^2$

$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial x}\frac{\partial x}{\partial t} = (a - 2bx) v_x = v_y$

$\frac{\partial^2 y}{\partial t^2} = (a-2bx)\frac{\partial v_x}{\partial t}^{\to 0} - 2b v_x^2$

$-2b v_x^2 = -g \quad \Rightarrow \quad v_x = \sqrt{\frac{g}{2b}}$

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{v_x^2 + \left(\frac{\partial y}{\partial x}\right)^2 v_x^2} = v_x\sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}$

$v = \sqrt{\left(\frac{g}{2b}\right)\left(1 + (a-2bx)^2\right)}$

at origin

$$v = \sqrt{\left(\frac{g}{2b}\right)\left(1 + a^2\right)}$$

Dis 6A 3.2

1.25) $y = at(1-\alpha t) \qquad a, \alpha > 0$

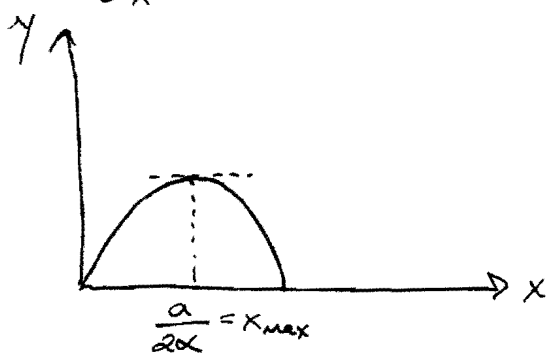
$x = at$

a) $y = x\left(1 - \frac{\alpha}{a}x\right) = x - \frac{\alpha}{a}x^2$

$\frac{\partial y}{\partial x} = 1 - \frac{\alpha}{a}2x = 0$ (max/min condition)

$\frac{a}{2\alpha} = x_{max/min}$

$\frac{\partial^2 y}{\partial x^2} = -2\frac{\alpha}{a}$ (concave down)




b) $\frac{\partial y}{\partial t} = a - 2a\alpha t = v_y \qquad \frac{\partial x}{\partial t} = a$

$|V| = \sqrt{a^2 + (a-2\alpha a t)^2}$

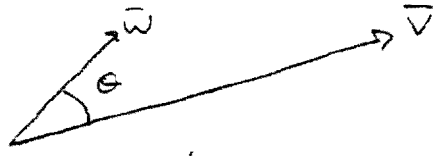
$V = a\hat{x} + (a-2\alpha a t)\hat{y}$

$\frac{\partial^2 y}{\partial t^2} = -2a\alpha = \omega_y \qquad \frac{\partial^2 x}{\partial t^2} = 0$

$\omega = -2a\alpha\,\hat{y}$

Dis. GA 3.3

1.25) c)




$\theta = \pi/4$

$$\frac{\bar{v}\cdot\bar{w}}{|w||v|} = \cos\theta$$

$$\frac{-2\alpha a(a-2\alpha at)}{2a\alpha\sqrt{a^2+(a-2a\alpha t)^2}} = \cos\theta$$

$$(2\alpha at-a)^2 = \cos^2\theta\left(a^2+(a-2a\alpha t)^2\right)$$

$$(1-\cos^2\theta)(2\alpha at-a)^2 = \cos^2\theta a^2$$

$$2\alpha at - a = \cot\theta\, a$$

$$t = \frac{a(1+\cot\theta)}{2\alpha a}$$

$$\boxed{t_0 = \frac{1}{\alpha}} \qquad \theta = \pi/4$$