1.3. LAWS OF CONSERVATION OF ENERGY, MOMENTUM, AND ANGULAR MOMENTUM

• Work and power of the force **F**:

$$A = \int \mathbf{F}\, d\mathbf{r} = \int F_s\, ds, \quad P = \mathbf{F}\mathbf{v}. \tag{1.3a}$$

• Increment of the kinetic energy of a particle:

$$T_2 - T_1 = A, \tag{1.3b}$$

where A is the work performed by the resultant of *all* the forces acting on the particle.

• Work performed by the forces of a field is equal to the decrease of the potential energy of a particle in the given field:

$$A = U_1 - U_2. \tag{1.3c}$$

• Relationship between the force of a field and the potential energy of a particle in the field:

$$\mathbf{F} = -\nabla U, \tag{1.3d}$$

i.e. the force is equal to the antigradient of the potential energy.

• Increment of the total mechanical energy of a particle in a given potential field:

$$E_2 - E_1 = A_{extr} \tag{1.3e}$$

where A_{extr} is the algebraic sum of works performed by all *extraneous* forces, that is, by the forces not belonging to those of the *given* field.

• Increment of the total mechanical energy of a system:

$$E_2 - E_1 = A_{ext} + A_{int}^{noncons}, \tag{1.3f}$$

where $E = T + U$, and U is the *inherent* potential energy of the system.

• Law of momentum variation of a system:

$$d\mathbf{p}/dt = \mathbf{F}, \tag{1.3g}$$

where **F** is the resultant of all *external* forces.

• Equation of motion of the system's centre of inertia:

$$m \frac{d\mathbf{v}_C}{dt} = \mathbf{F}, \tag{1.3h}$$

where **F** is the resultant of all *external* forces.

• Kinetic energy of a system

$$T = \widetilde{T} + \frac{mv_C^2}{2}, \tag{1.3i}$$

where $\widetilde{T}$ is its kinetic energy in the system of centre of inertia.

• Equation of dynamics of a body with variable mass:

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F} + \frac{dm}{dt}\mathbf{u}, \tag{1.3j}$$

where **u** is the velocity of the separated (gained) substance relative to the body considered.

• Law of angular momentum variation of a system:

$$\frac{d\mathbf{M}}{dt} = \mathbf{N}, \tag{1.3k}$$

where **M** is the angular momentum of the system, and **N** is the total moment of all *external* forces.

• Angular momentum of a system:

$$\mathbf{M} = \widetilde{\mathbf{M}} + [\mathbf{r}_C\mathbf{p}], \tag{1.3l}$$

where $\widetilde{\mathbf{M}}$ is its angular momentum in the system of the centre of inertia, $\mathbf{r}_C$ is the radius vector of the centre of inertia, and **p** is the momentum of the system.

• **1.118.** A particle has shifted along some trajectory in the plane xy from point *1* whose radius vector $\mathbf{r}_1 = \mathbf{i} + 2\mathbf{j}$ to point *2* with the radius vector $\mathbf{r}_2 = 2\mathbf{i} - 3\mathbf{j}$. During that time the particle experienced the action of certain forces, one of which being $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j}$. Find the work performed by the force **F**. (Here r_1, r_2, and F are given in SI units).

• **1.119.** A locomotive of mass m starts moving so that its velocity varies according to the law $v = a\sqrt{s}$, where a is a constant, and s is the distance covered. Find the total work performed by all the forces which are acting on the locomotive during the first t seconds after the beginning of motion.

•• **1.120.** The kinetic energy of a particle moving along a circle of radius R depends on the distance covered s as $T = as^2$, where a is a constant. Find the force acting on the particle as a function of s.

••• **1.121.** A body of mass m was slowly hauled up the hill (Fig. 1.29) by a force **F** which at each point was directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is h, the length of its base l, and the coefficient of friction k.

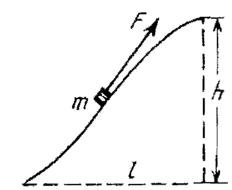

Fig. 1.29.

•• **1.122.** A disc of mass $m = 50$ g slides with the zero initial velocity down an inclined plane set at an angle $\alpha = 30°$ to the horizontal; having traversed the distance $l = 50$ cm along the horizontal plane, the disc stops. Find the work performed by the friction forces over the whole distance, assuming the friction coefficient $k = 0.15$ for both inclined and horizontal planes.

• **1.123.** Two bars of masses m_1 and m_2 connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between the bars and the surface is equal to k. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar?

1.124. A chain of mass $m = 0.80$ kg and length $l = 1.5$ m rests on a rough-surfaced table so that one of its ends hangs over the edge. The chain starts sliding off the table all by itself provided the overhanging part equals $\eta = 1/3$ of the chain length. What will be the