what value of the angle α will the time of sliding be the least? What will it be equal to?

•• **1.67.** A bar of mass m is pulled by means of a thread up an inclined plane forming an angle α with the horizontal (Fig. 1.13). The coefficient of friction is equal to k. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be minimum. What is it equal to?

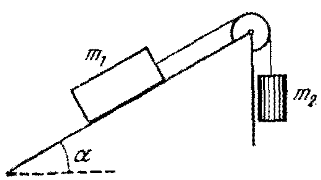

Fig. 1.11.

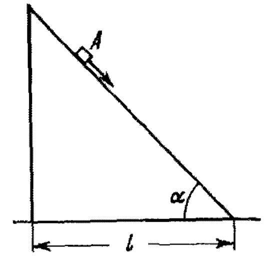

Fig. 1.12.

•• **1.68.** At the moment $t = 0$ the force $F = at$ is applied to a small body of mass m resting on a smooth horizontal plane (a is a constant). The permanent direction of this force forms an angle α with the horizontal (Fig. 1.14). Find:

(a) the velocity of the body at the moment of its breaking off the plane;

(b) the distance traversed by the body up to this moment.

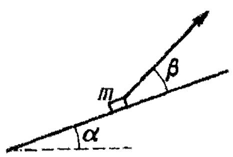

Fig. 1.13.

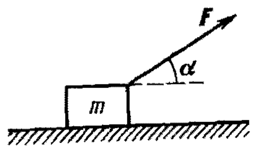

Fig. 1.14.

•• **1.69.** A bar of mass m resting on a smooth horizontal plane starts moving due to the force $F = mg/3$ of constant magnitude. In the process of its rectilinear motion the angle α between the direction of this force and the horizontal varies as $\alpha = as$, where a is a constant, and s is the distance traversed by the bar from its initial position. Find the velocity of the bar as a function of the angle α.

•• **1.70.** A horizontal plane with the coefficient of friction k supports two bodies: a bar and an electric motor with a battery on a block. A thread attached to the bar is wound on the shaft of the electric motor. The distance between the bar and the electric motor is equal to l. When the motor is switched on, the bar, whose mass is twice as great as that of the other body, starts moving with a constant acceleration w. How soon will the bodies collide?

••• **1.71.** A pulley fixed to the ceiling of an elevator car carries a thread whose ends are attached to the loads of masses m_1 and m_2. The car starts going up with an acceleration $\mathbf{w}_0$. Assuming the masses of the pulley and the thread, as well as the friction, to be negligible find:

(a) the acceleration of the load m_1 relative to the elevator shaft and relative to the car;

(b) the force exerted by the pulley on the ceiling of the car.

•• **1.72.** Find the acceleration $\mathbf{w}$ of body 2 in the arrangement shown in Fig. 1.15, if its mass is η times as great as the mass of bar 1 and the angle that the inclined plane forms with the horizontal is equal to α. The masses of the pulleys and the threads, as well as the friction, are assumed to be negligible. Look into possible cases.

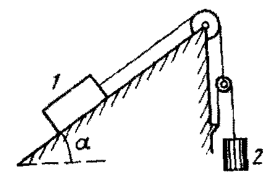

Fig. 1.15.

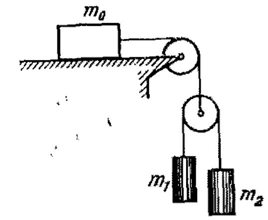

Fig. 1.16.

•• **1.73.** In the arrangement shown in Fig. 1.16 the bodies have masses m_0, m_1, m_2, the friction is absent, the masses of the pulleys and the threads are negligible. Find the acceleration of the body m_1. Look into possible cases.

•• **1.74.** In the arrangement shown in Fig. 1.17 the mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting it to slide along the thread with some friction. The mass of the pulley and the friction in its axle are negligible. At the initial moment the ball was located opposite the lower end of the rod. When set free,

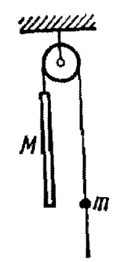

Fig. 1.17.

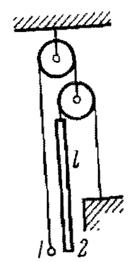

Fig. 1.18.

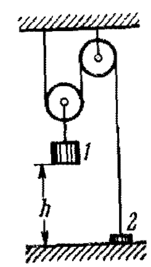

Fig. 1.19.

1.3. LAWS OF CONSERVATION OF ENERGY, MOMENTUM, AND ANGULAR MOMENTUM

• Work and power of the force **F**:

$$A = \int \mathbf{F}\, d\mathbf{r} = \int F_s\, ds, \quad P = \mathbf{F}\mathbf{v}. \qquad (1.3a)$$

• Increment of the kinetic energy of a particle:

$$T_2 - T_1 = A, \qquad (1.3b)$$

where A is the work performed by the resultant of *all* the forces acting on the particle.

• Work performed by the forces of a field is equal to the decrease of the potential energy of a particle in the given field:

$$A = U_1 - U_2. \qquad (1.3c)$$

• Relationship between the force of a field and the potential energy of a particle in the field:

$$\mathbf{F} = -\nabla U, \qquad (1.3d)$$

i.e. the force is equal to the antigradient of the potential energy.

• Increment of the total mechanical energy of a particle in a given potential field:

$$E_2 - E_1 = A_{extr} \qquad (1.3e)$$

where A_{extr} is the algebraic sum of works performed by all *extraneous* forces, that is, by the forces not belonging to those of the *given* field.

• Increment of the total mechanical energy of a system:

$$E_2 - E_1 = A_{ext} + A_{int}^{noncons}, \qquad (1.3f)$$

where $E = T + U$, and U is the *inherent* potential energy of the system.

• Law of momentum variation of a system:

$$d\mathbf{p}/dt = \mathbf{F}, \qquad (1.3g)$$

where **F** is the resultant of all *external* forces.

• Equation of motion of the system's centre of inertia:

$$m\frac{d\mathbf{v}_C}{dt} = \mathbf{F}, \qquad (1.3h)$$

where **F** is the resultant of all *external* forces.

• Kinetic energy of a system

$$T = \widetilde{T} + \frac{mv_C^2}{2}, \qquad (1.3i)$$

where $\widetilde{T}$ is its kinetic energy in the system of centre of inertia.

• Equation of dynamics of a body with variable mass:

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F} + \frac{dm}{dt}\mathbf{u}, \qquad (1.3j)$$

where **u** is the velocity of the separated (gained) substance relative to the body considered.

• Law of angular momentum variation of a system:

$$\frac{d\mathbf{M}}{dt} = \mathbf{N}, \qquad (1.3k)$$

where **M** is the angular momentum of the system, and **N** is the total moment of all *external* forces.

• Angular momentum of a system:

$$\mathbf{M} = \widetilde{\mathbf{M}} + [\mathbf{r}_C \mathbf{p}], \qquad (1.3l)$$

where $\widetilde{\mathbf{M}}$ is its angular momentum in the system of the centre of inertia, $\mathbf{r}_C$ is the radius vector of the centre of inertia, and **p** is the momentum of the system.

• **1.118.** A particle has shifted along some trajectory in the plane xy from point *1* whose radius vector $\mathbf{r}_1 = \mathbf{i} + 2\mathbf{j}$ to point *2* with the radius vector $\mathbf{r}_2 = 2\mathbf{i} - 3\mathbf{j}$. During that time the particle experienced the action of certain forces, one of which being $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j}$. Find the work performed by the force **F**. (Here r_1, r_2, and F are given in SI units).

• **1.119.** A locomotive of mass m starts moving so that its velocity varies according to the law $v = a\sqrt{s}$, where a is a constant, and s is the distance covered. Find the total work performed by all the forces which are acting on the locomotive during the first t seconds after the beginning of motion.

•• **1.120.** The kinetic energy of a particle moving along a circle of radius R depends on the distance covered s as $T = as^2$, where a is a constant. Find the force acting on the particle as a function of s.

••• **1.121.** A body of mass m was slowly hauled up the hill (Fig. 1.29) by a force **F** which at each point was directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is h, the length of its base l, and the coefficient of friction k.

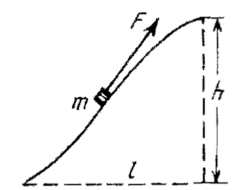

Fig. 1.29.

•• **1.122.** A disc of mass $m = 50$ g slides with the zero initial velocity down an inclined plane set at an angle $\alpha = 30°$ to the horizontal; having traversed the distance $l = 50$ cm along the horizontal plane, the disc stops. Find the work performed by the friction forces over the whole distance, assuming the friction coefficient $k = 0.15$ for both inclined and horizontal planes.

• **1.123.** Two bars of masses m_1 and m_2 connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between the bars and the surface is equal to k. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar?

1.124. A chain of mass $m = 0.80$ kg and length $l = 1.5$ m rests on a rough-surfaced table so that one of its ends hangs over the edge. The chain starts sliding off the table all by itself provided the overhanging part equals $\eta = 1/3$ of the chain length. What will be the