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Emergent Relativity

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A possible resolution of the incompatibility of quantum mechanics and general relativity is that the relativity principle is emergent. I show that the central paradox of black holes also occurs at a liquid-vapor critical surface of a bose condensate but is resolved there by the phenomenon of quantum criticality. I propose that real black holes are actually phase boundaries of the vacuum analogous to this, and that the Einstein field equations simply fail at the event horizon the way quantum hydrodynamics fails at a critical surface. This can occur without violating classical general relativity anywhere experimentally accessible to external observers. Since the low-energy effects that occur at critical points are universal, it is possible to make concrete experimental predictions about such surfaces without knowing much, if anything about the true underlying equations. Many of these predictions are different from accepted views about black holes - in particular the absence of Hawking radiation and the possible transparency of cosmological black hole surfaces. [To appear in the C. N Yang Festschrift (World Sci., Singapore, 2003).]

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I. INTRODUCTION

It is a great honor for me to be speaking at this symposium for Prof. C. N. Yang. Like many other physicists, I have always envied Prof. Yang's many excellent contributions to science over the years, and have even shared the common experience of aspiring to match them and managing to fall short. Dealing with this is not a happy thing. I confess having become depressed over it for a time and cheering up only after realizing that everyone else had the same problem. I now no longer worry about it. One should no more agonize over this inadequacy than over being too short or bald. I recommend this course of action for the rest of you sufferers, incidentally, in case you have not figured it out already for yourselves. I also recommend that we keep trying, for Prof. Yang continues to be the man to beat.

My views on the great unsolved questions at the core of modern physics—quantum measurement, the emergence of the correspondence limit through decoherence, spontaneous ordering, hierarchies of laws—are strongly influenced by my life in condensed matter physics, where theoretical ideas are forced to immediate and brutal confrontation with experiment by virtue of the latter's low cost. Anyone subjected to this long enough eventually develops the habit of thinking experimentally, of choosing experimental issues primarily on the basis of what one could measure in a given situation, and evaluating theories mainly on the basis of the experiments they correctly predict. This is considered overly conservative in many circles, but I disagree. I believe that physics is an experimental science, and that theory acquires authority by confronting and conforming to experiment, not the other way around. Dealing with a rich experimental record day after day has the additional benefit of giving one a healthy respect for the natural world's ability to surprise and a healthy *dis*respect for the belief that all things can be calculated from first principles.

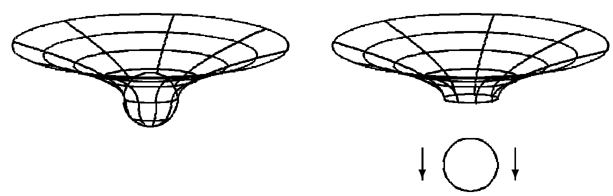

FIG. 1: Einstein gravity is similar to a heavy ball placed on a rubber membrane, except that the membrane ruptures if the ball is too heavy. Rupturing can be prevented by declaring the laws of elasticity to be true no matter how extreme the stretching, but this is unphysical. The solution to the black hole problem may be that the relativity principle, like elasticity, is emergent and simply fails at the event horizon.

II. THE RELATIVITY PRINCIPLE

I wish today to discuss the black hole horizon paradox and the incompatibility of relativity and quantum mechanics. This is obviously a great problem in the physics pantheon and something of great interest to all of us, particularly in light of recent advances in string theory. However, what I have to say is not so friendly to microscopic approaches of this kind. I have become increasingly concerned that the essence of the problem may not be microscopic at all but collective, and that studying microscopic models of the vacuum may be the wrong thing to do *even if the models are right*. I think black hole formation may be a quantum phase transition[1,2].

Before explaining how such a thing could be consistent with Einstein gravity and working out the experimental consequences, let me explain the basic idea, which is straightforward. Let us imagine a stretched rubber sheet with a heavy ball rolling on it, as shown in Fig. 1. This is the stripped-down model of gravity familiar from science museum exhibits. The membrane represents space-time, the ball represents some gravitating object, the distortion represents the gravitational field, and the motions of small objects on the membrane represent geodesic trajectories of satellites. If the ball is not too heavy then the membrane distorts elastically to make a slight depression in response to its weight. Small objects in the vicinity then fall into this distortion and orbit, membrane vibrations beamed at the ball scatter, and so forth in analogy with general relativity. If the ball is too heavy, on the other hand, the membrane ruptures and the ball falls through. When this happens the analogy with Einstein gravity fails completely, since the relativity principle requires every point in space-time to be locally indistinguishable from any other, and thus expressly forbids rupture. The formal statement of this problem is that the black hole event horizon, the obvious candidate for catastrophic failure in general relativity, exhibits no singular behavior in any quantity measured in local coordinates. However, catastrophic failure is precisely what I suspect is happening at real black hole surfaces.

What would facilitate this breakdown and at the same time reconcile it with what we know experimentally about relativity are the limit paradoxes of continuous quantum phase transitions[3,4]. Known physical principles operating at such transitions would enable relativity to fail quantum mechanically, just as laws of elasticity fail in the membrane, but so gently that classical Einstein gravity would *not* be violated in any region of space-time experimentally accessible to us[1]. This latter point is not obvious, especially if one has not thought carefully about phases and phase transitions, so one of my main tasks here today is to explain it convincingly.

The possibility of relativity failure is difficult for most of us to think about, but should not be. When asked whether relatively applies at the Planck scale, for example, most physicists will attempt to change the subject and, in the end, avoid committing one way or the other even though traditional relativity forbids preferred scales. Everybody understands that relativity is believable because it is measured to be true, not because it ought to be true, and that extrapolating many orders of magnitude beyond present measurement capability is no less dangerous in relativity than it is in anything else.

III. CRITICAL OPALESCENCE

Let me now begin by reminding you about the simplest known example of a phase transition, ordinary vaporization. In 1910 Johannes van der Waals won the Nobel Prize in physics for his work on non-ideal gasses, and particularly for his invention of the relation

$$(p + \frac{a}{v^2})(v - b) = k_B T \quad , \tag{1}$$

known today as the van der Waals equation of state[5]. This approximate description of the non-deal gas captures the essential features of both departures from ideality in real vapors and the liquid-vapor transition they

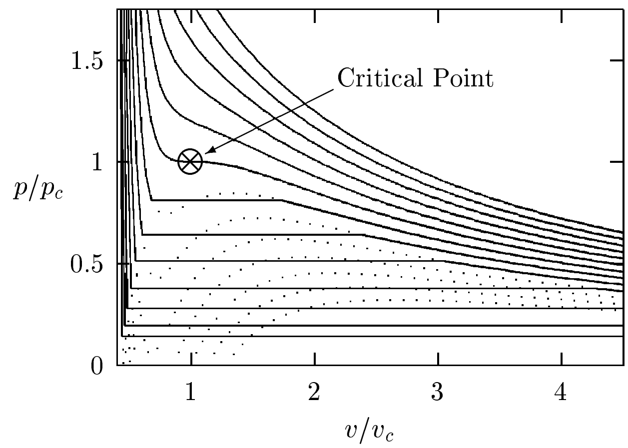

FIG. 2: Pressure versus volume per atom given by Eq. (1) for various temperatures. The critical volume, pressure and temperature are $v_c = 3b$, $p_c = a/27b^2$, and $k_B T_c = 8a/27b$. The increment between successive isotherms is $\Delta T = 0.05 T_c$. The dotted lines show the equation of state before the Maxwell construction. For freon (CCl_2F_2) the critical temperature is 385 °K and the critical pressure 4.12 MPa (37 atmospheres).

anticipate. p is the pressure, v is the volume per molecule, T is the temperature, k_B is Boltzmann's constant, and a and b are parameters characterizing the non-ideality of the fluid. a represents the effects of attractive bonding forces between the molecules, and b represents the volume excluded due to short-range molecular repulsions. Both parameters are empirically adjusted to fit the properties of a specific substance. Setting them to zero produces the ideal gas law.

The van der Waals equation of state describes the phase transition only implicitly. It may be seen in Fig. 2 that its isotherms exhibit unphysical inflections at low temperatures that result in the bulk modulus

$$k = -v \frac{\partial p}{\partial v} \tag{2}$$

becoming negative. This is a symptom of the theory's failure to correctly describe liquid-vapor coexistence. From the vast amount of work done on this problem in the 1970s, culminating in the invention of the Wilson renormalization group[6], we understand that equations of state near phase transitions are inherently nonanalytic, and that analytic fits to them generally produce nonsense when extrapolated across phase boundaries. These non-analyticities are, however, effects of large size and disappear when the sample is small. Taken literally, the Van der Waals equation of state is a description of a small sample. To apply it to a large sample we must take into account for the system's tendency to separate into regions of high and low density. This is accomplished by finding a pressure at which the area under a straight line drawn between the two extremal volumes is the same as that under the inflecting equation of state. The end points of this Maxwell construction then define the liquid and vapor densities[5].

The critical point - the top of the liquid-vapor dome where the two-phase region shrinks to zero - is especially important for our discussion of black holes. At this point, and this point only, the bulk modulus of the fluid is identically zero. For temperatures above the critical temperature T_c, inflection does not occur, the liquid and vapor phases are physically indistinguishable, and the bulk modulus is positive. For temperature less than T_c, phase separation occurs, and the bulk modulus at either end of the Maxwell construction is again positive. Thus at this point, and this point only, the speed of sound

$$c = \sqrt{k/\rho} \quad , \tag{3}$$

where M and $\rho = M/v$ are the mass and mass density of the molecules, vanishes.

In conventional fluids, the vanishing of the sound speed at the critical point causes critical opalescence, a strange phenomenon in which the fluid becomes cloudy and opaque to the transmission of light, like an opal. This is very dramatic to see. My colleague Doug Osheroff here at Stanford has a freshman physics demonstration of critical opalescence that uses freon as the working fluid. The critical pressure and temperature of freon are sufficiently low that one can do this without endangering students in the front row. Doug sets the temperature to the critical value and then slowly ramps up the pressure, while Richard Strauss's *Also Sprach Zarathustra* plays in the background. If he times it right, the laser shining through the freon winks out at the exact moment the orchestra plays "Ta Daa," and gets him a standing ovation.

Critical opalescence signals a fundamental failure of hydrodynamics[4]. Equipartition among the degrees of freedom of compressional sound, which exist by virtue of the principles of hydrodynamics, requires the departure of the particle density from its average value to obey

$$< \delta\rho(\mathbf{r}) \delta\rho(\mathbf{r}') > = \frac{k_B T \, \rho^2}{(2\pi)^3 k} \int_{|\mathbf{q}|<1/\xi} e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \, d\mathbf{q} \quad , \tag{4}$$

where $\mathbf{r}$ and $\mathbf{r}'$ denote different positions in the fluid, $1/\xi$ is an ultraviolet cutoff (i.e. a scale at which hydrodynamics fails), and $<>$ denotes thermal average. This is the quantity measured in a light scattering experiment. It becomes enormous at the critical point because k goes to zero. However, since the density correlator cannot actually become infinite, we know that the key premise of the calculation, the validity of hydrodynamics, must fail. This occurs in practice through the divergence of ξ at the critical point.

The laws of hydrodynamics are emergent. They are universal, exact mathematical relationships among measured quantities that develop at long length scales in liquids and gases. This development cannot be deduced from the underlying equations of motion of the atoms.

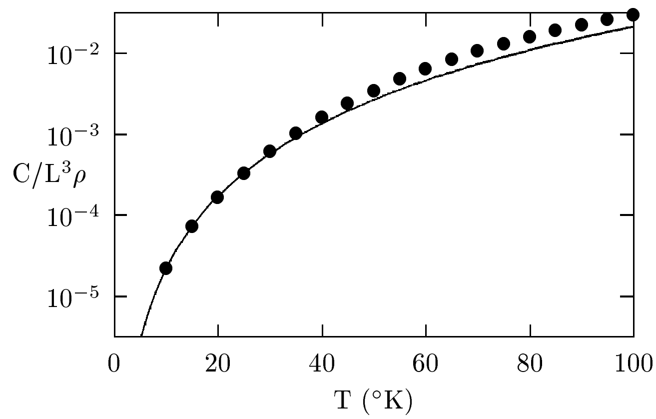

FIG. 3: Specific heat of Al_2O_3 in units of calories/gm °K versus temperature as measured[7] (dots) and as given by Eq. (5) with c = 6142 m/sec. The mass density, bulk modulus and poisson ratio of this material are $\rho = 3.89$ gm/cm^3, $k = 228$ GPa, and $\sigma = 0.22$, respectively. The transverse and longitudinal sound speeds computed from these are then $c_t = 6355$ m/sec and $c_\ell = 10602$ m/sec. Their appropriately weighted average is $2c^{-3} = 2c_t^{-3} + c_\ell^{-3}$.

It is a physical phenomenon - one we know to be exactly true because it is *measured* to be true. Emergent laws are equivalent to, and indistinguishable from, fundamental laws in all ways but one: they are vulnerable to failure by simply not emerging. This is what happens at the critical point.

IV. QUANTUM PHASES

Phases and phase transitions are not inherently finite-temperature phenomena. They occur at zero temperature as well, and are regulated by principles of emergence in the same way as their finite-temperature relatives. The important difference is that zero-temperature phases are purely quantum-mechanical phenomena.

A quantum phase familiar from everyday experience is the crystalline solid. If this is an insulator its low-energy quantum excitations consist of center-of-mass motion and sound solely. Both are quantum-mechanical. The sound waves of the cold solid are quantized "particles" with apt physical similarities with particles of light. This analogy becomes increasingly exact as the energy scale is lowered and, in the end, results in the low-temperature specific heat of all crystalline insulators becoming the Planck blackbody law

$$\frac{C}{L^3} = \frac{4\pi^2}{15} (\frac{k_B T}{\hbar c})^3 \, k_B \tag{5}$$

with the speed of light rescaled down to the speed of sound. This is shown for the specific case of Al_2O_3 in Fig. 3.

The liquid and gas phases also exist at zero temperature. The liquid is realized by either ^{4}He nor ^{3}He—

both of which become superfluids when cooled to zero temperature (although ^{3}He is more complicated)[8]. The compressional sound waves in these fluids are quantum-mechanical particles that become more and more accurately defined and relativistic as the energy scale is lowered. Their vapor pressures become unmeasurably small at low temperatures—meaning that both will puddle at the bottom of a container and will not expand to fill the available volume. The vapor is realized by atomic bose-einstein condensates, the discovery of which by Cornell, Ketterle, and Weiman was awarded the Nobel Prize in physics in 2001[9,10]. Atomic condensates are metastable states of matter and thus not, strictly speaking, quantum phases. However, this is unimportant. They are ground states of an equivalent fictitious hamiltonian and have all the important physical properties of phases. Like ^{4}He and ^{3}He they exist as superfluids at ultralow temperatures and have nonzero bulk moduli[12]. Unlike helium, however, they *do* expand to fill any available volume.

The existence of the quantum liquid and gas in nature means that we can think about the phase transition between them, even though it has never been observed in the laboratory. The ideal behavior would be a phase diagram something like Fig. 2 except with the "temperature" reinterpreted as parameter in the underlying equations of motion.

Unfortunately, not every parameterization of this transition produces a phase diagram like that of Fig. 2. Real ^{4}He is described by the equations

$$i\hbar \frac{\partial \Psi}{\partial t} = \mathcal{H}\ \Psi \tag{6}$$

$$\mathcal{H} = -\sum_j^N \frac{\hbar^2}{2M} \nabla_j^2 + V(\mathbf{r}_1, ..., \mathbf{r}_N) \tag{7}$$

$$V(\mathbf{r}_1, ..., \mathbf{r}_N) = \sum_{j<k} V_{\rm pair}(\mathbf{r}_j - \mathbf{r}_k) \quad , \tag{8}$$

where $V_{\rm pair}$ is a pair potential, given reasonably accurately by

$$V_{\rm pair}(\mathbf{r}) = V_0 \left[(\frac{r_0}{r})^{12} - 2(\frac{r_0}{r})^6 \right] \quad , \tag{9}$$

with $r_0 = 3$ Å and $V_0 = 11$ °K[13]. We know this to be true because the pair potential has been accurately measured in atomic beam experiments (*cf.* Fig. 4). Also, variational calculations using this potential predict the correct ground state energy[14], pair correlation function, crystallization pressure and superfluid transition temperature[15]. Since the cohesion of the liquid comes from the coefficient of of $1/r^6$ in this potential, the most obvious way to produce the gas phase is to reduce this

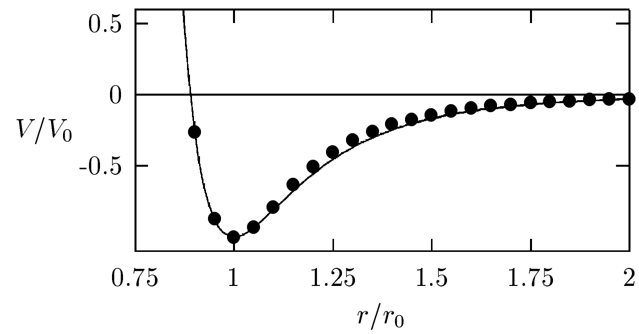

FIG. 4: Comparison of He-He pair potential measured by atomic-beam scattering[11] (dots) with the model of Eq. (9).

coefficient to zero. However extensive computer modeling in the 1970s showed that the transition so generated is strongly first-order and cannot be tuned to criticality with volume[16].

V. SUPERFLUIDITY

To make a phase diagram like that of Fig. 2 we must add multi-body interaction potentials[17]. The simplest way to accomplish this is to sum short-range components into a density functional. Let us consider specifically

$$V(\mathbf{r}_1, ..., \mathbf{r}_N) = V_1 N + V_2 \sum_{j<k}^N (\frac{\beta r_0^2}{\pi})^{3/2} e^{-\beta |\mathbf{r}_j - \mathbf{r}_k|^2} + ...$$

$$+ V_n \sum_{j_1 < ... < j_n} n^{-3/2} \left[\frac{n \beta r_0^2}{\pi} \right]^{(3n-3)/2} \prod_{\mu < \nu} e^{-\beta |\mathbf{r}_{j_\mu} - \mathbf{r}_{j_\nu}|^2} \tag{10}$$

where r_0 is a characteristic length, in the limit $\beta \to \infty$. The multi-center functions in this expression are zero unless n particles are coincident in space, and are normalized to yield 1 when integrated on all but one of their arguments. With this convention Eqs. (6) and (7) may be re-expressed compactly as the classical lagrangian density[18,19]

$$\mathcal{L} = \psi^* (i\hbar \frac{\partial}{\partial t} + \mu) \psi - \frac{\hbar^2}{2M} |\nabla \psi|^2 - \mathcal{U}(|\psi(\mathbf{r}|^2) \quad , \tag{11}$$

where μ is the chemical potential and

$$\mathcal{U}(|\psi|^2) = \frac{V_1}{1!} |\psi|^2 + \frac{V_2}{2!} r_0^3 |\psi|^4 + \frac{V_3}{3!} r_0^6 |\psi|^6 + ... \ . \tag{12}$$

Thus $V_j r_0^{3(j-1)}$ are simply the Taylor expansion coefficients of $\mathcal{U}$.

The quantum equation of state implicit in models such as Eq. (11) are easy to work out when the potentials are weak[20]. The reason is that the fluid bose condenses, the operator ψ acquires a vacuum expectation value $\psi \rightarrow <\psi>$, and the entire problem becomes classical. As with any superfluid order parameter, the square ψ has the physical meaning of a particle density: $|\psi|^2 = 1/v$. The usual rules of canonical quantization then give us for the expected energy

$$<\mathcal{H}> = \int \left[\frac{\hbar^2}{2M} |\nabla\psi|^2 + \mathcal{U}(|\psi|^2) - \mu|\psi|^2 \right] d\mathbf{r} \quad . \tag{13}$$

This allows us to identify $\mathcal{U}(|\psi|^2)$ as the energy per unit volume of the fluid as a function of its density, and

$$p = |\psi|^2 \mathcal{U}'(|\psi|^2) - \mathcal{U}(|\psi|^2) \tag{14}$$

as the pressure. The quantum ground state is implicitly defined by the energy-minimization condition

$$\mu = \mathcal{U}'(|\psi|^2) \quad . \tag{15}$$

It is also easy to work out what $\mathcal{U}$ must be to give an equation of state like that of Fig. 2. We need to modify Eq. (1) somewhat because the pressure of a quantum fluid is due to interactions and thus must fall off at least as fast as $1/v^2$ if the potentials are to be short-ranged. Thus we take

$$(p + \frac{a}{v^4})(v^2 - b) = d \quad , \tag{16}$$

where a, b, and d are hamiltonian parameters. Since

$$\int p \, dv = \frac{d}{2\sqrt{b}} \ln(\frac{v - \sqrt{b}}{v + \sqrt{b}}) + \frac{a}{3v^3} \quad , \tag{17}$$

we then have

$$\mathcal{U}(|\psi|^2) = d - \frac{d}{2\sqrt{b}|\psi|^2} \ln(\frac{1 - \sqrt{b}|\psi|^2}{1 + \sqrt{b}|\psi|^2}) - \frac{a}{3}|\psi|^4$$

$$= -\frac{a}{3}|\psi|^4 + d\left[\frac{(\sqrt{b}|\psi|^2)^2}{3} + \frac{(\sqrt{b}|\psi|^2)^4}{5} + ... \right] \quad . \tag{18}$$

The analysis of this equation of state, including the Maxwell construction, is exactly the same as with Eq. (1). It becomes critical when $d = 8a/27b$.

The dispersion relation of compressional sound implicit in such models has an important characteristic form. Because it is classical, the motion is defined by the extremal condition $\delta\mathcal{L}/\delta\psi^* = 0$, which is satisfied when

$$i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2M} \nabla^2\psi + \left[\mathcal{U}'(|\psi|^2) - \mu \right] \psi \quad . \tag{19}$$

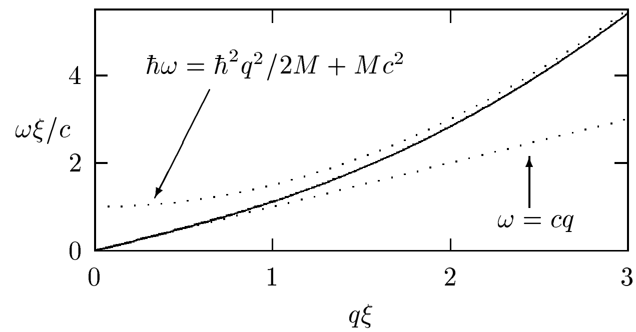

FIG. 5: Dispersion relation given by Eq. (22). The dotted lines are asymptotic behaviors at large and small q. As its momentum increases this excitation evolves adiabatically from a relativistic sound quantum into a free boson.

Assuming that $\psi = \psi_0 + \delta\psi_R + i\delta\psi_I$, where ψ_0 is real and $\delta\psi_R$ and $\delta\psi_I$ are both small, we have, to linear order,

$$\hbar \frac{\partial(\delta\psi_R)}{\delta t} = -\frac{\hbar^2}{2M} \nabla^2(\delta\psi_I)$$

$$-\hbar \frac{\partial(\delta\psi_I)}{\delta t} = -\frac{\hbar^2}{2M} \nabla^2(\delta\psi_R) + 2\mathcal{U}''(\psi_0^2)\psi_0^2 \, \delta\psi_R \quad . \tag{20}$$

Then substituting $\delta\psi_R = \delta\psi_R^{(0)} e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)}$ and $\delta\psi_I = \delta\psi_I^{(0)} e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)}$, and noting that

$$\mathcal{U}''(\psi_0^2) \, \psi_0^2 = kv \quad , \tag{21}$$

we obtain for the dispersion relation

$$\hbar\omega_q = \sqrt{(\hbar cq)^2 + (\frac{\hbar^2 q^2}{2M})^2} \qquad (\, c = \sqrt{\frac{k}{\rho}} \,) \quad . \tag{22}$$

This is plotted in Fig. 5.

Eq. (22) contains a length scale

$$\xi = \hbar/Mc \quad . \tag{23}$$

central to our discussion. The linear relation $\omega = cq$ expected of a compressional fluid occurs only when $q\xi << 1$. At scales longer than ξ, the principles of quantum hydrodynamics, the zero-temperature version of the familiar laws of classical fluids, become exact, and we obtain a gas of noninteracting relativistic scalar bosons characterized by velocity c. At length scales shorter than ξ, on the other hand, the principles of quantum hydrodynamics fail, and these particles acquire a decay width and are not guaranteed even to exist.

VI. QUANTUM ENTANGLEMENT

Eq. (22) is extremely important for our argument, so let us derive it by a second method due to Bogoliubov[21,22]. The reasoning in this case is more straightforward but less general, in that it applies only to the weakly-interacting bose gas.

The starting point of the calculation is again conventional quantum mechanics as defined by Eqs. (6) and (7), but we use only a pair sum, as in Eq. (8), with the short-range repulsive potential

$$V_{\rm pair}({\bf r}) = V_2 r_0^3 \; \delta^3({\bf r}) \quad . \tag{24}$$

With this done Eq. (7) may be rewritten

$$\mathcal{H} = \sum_{\bf q} \frac{\hbar^2 q^2}{2M} \; a_{\bf q}^\dagger a_{\bf q}$$

$$+ \frac{V_2 r_0^3}{2L^3} \sum_{\bf q} \sum_{{\bf q}'} \sum_{\Delta{\bf q}} a_{{\bf q}+\Delta{\bf q}}^\dagger a_{{\bf q}'-\Delta{\bf q}}^\dagger a_{{\bf q}'} a_{\bf q} \quad , \tag{25}$$

where $a_{\bf q}^\dagger$ and $a_{\bf q}$ are bose creation and annihilation operators satisfying the usual commutation relations

$$[a_{\bf q}, a_{{\bf q}'}] = 0 \qquad [a_{\bf q}, a_{{\bf q}'}^\dagger] = \delta_{{\bf q}{\bf q}'} \quad . \tag{26}$$

We assume for simplicity that the bosons live in a box of volume L^3 with periodic boundary conditions and that the pair potential is weak.

The calculation is again simplified by the phenomenon of bose condensation. This causes the ${\bf q} = 0$ state to acquire macroscopic occupancy N. When this number is thermodynamically large (but not otherwise) the ${\bf q} = 0$ state becomes a particle reservoir for the rest of the system. The most significant effect of the potential is to then to scatter particles out of, and back into, the condensate in pairs. The matrix element for this process is always about $N V_2 r_0^3 / L^3$, since $< a_0^\dagger a_0 > \simeq N$. Thus if we ignore the depletion of N due to scattering of bosons out of the condensate then we may simply replace a_0 and $a_0^\dagger$ everywhere they appear in Eq. (25) with $\sqrt{N}$. This gives

$$\mathcal{H}_{\rm eff} = \sum_{{\bf q}\neq 0} \biggl\{ (\frac{\hbar^2 q^2}{2M} + \frac{N V_2 r_0^3}{L^3}) \; a_{\bf q}^\dagger a_{\bf q}$$

$$+ \frac{N V_2 r_0^3}{2L^3} (a_{\bf q}^\dagger a_{-{\bf q}}^\dagger + a_{-{\bf q}} a_{\bf q}) \biggr\} + \frac{N^2 V_2 r_0^3}{2L^3} \quad . \tag{27}$$

$\mathcal{H}_{\rm eff}$ does not conserve particle number, but this is simply an approximation. In reality any promotion of particles out of the condensate will be matched by a corresponding reduction of N.

This hamiltonian may be easily diagonialized by canonical transformation. Let

$$b_{\bf q} = u_q a_{\bf q} + v_q a_{-{\bf q}}^\dagger \quad . \tag{28}$$

Then the condition

$$[b_{\bf q}, b_{{\bf q}'}] = 0 \qquad [b_{\bf q}, b_{{\bf q}'}^\dagger] = \delta_{{\bf q}{\bf q}'} \tag{29}$$

requires that $u_q^2 - v_q^2 = 1$, which is satisfied when

$$u_q = \cosh(\theta_q) \qquad v_q = \sinh(\theta_q) \quad . \tag{30}$$

The inverse of Eq. (29) is $a_{\bf q} = u_q b_{\bf q} - v_q b_{-{\bf q}}^\dagger$. Substituting this into Eq. (27) we find that the coefficients of $b_{\bf q}^\dagger b_{-{\bf q}}^\dagger$ and $b_{-{\bf q}} b_{\bf q}$ vanish provided that

$$\frac{u_q^2 + v_q^2}{2 v_q u_q} = \frac{1}{\tanh(2\theta_q)} = 1 + \frac{L^3}{N V_2 r_0^3} \; \frac{\hbar^2 q^2}{2M} \quad . \tag{31}$$

With this choice of u_q and v_q we obtain finally

$$\mathcal{H}_{\rm eff} = \sum_{\bf q} \biggl\{ \hbar \omega_q \; b_{\bf q}^\dagger b_{\bf q} + \frac{\hbar^2 q^2}{2M} v_q^2 - \frac{\hat{N} V_2 r_0^3}{L^3} u_q v_q \biggr\}$$

$$+ \frac{\hat{N} V_2 r_0^2}{2L^3} \qquad (\; \hat{N} = N + \sum_{\bf q} v_q^2 \;) \; , \tag{32}$$

where $\hbar\omega_q$ is given by Eq. (22) with

$$M c^2 = \frac{N V_2 r_0^3}{L^3} \quad . \tag{33}$$

The ground state implicit in this solution is a highly “entangled” state in which the number of bosons with momentum ${\bf q}$ is correlated with the number at $-{\bf q}$. From the condition that every $b_{\bf q}$ annihilate the ground state $|\Psi>$ we find that

$$|\Psi> = \exp(- \sum_{\bf q} \frac{v_q}{2u_q} a_{\bf q}^\dagger a_{-{\bf q}}^\dagger) (a_0^\dagger)^N |0> \quad . \tag{34}$$

The expressions for u_q and v_q may also be obtained by adopting a ground state of this form as a variational ansatz and minimizing the expected energy with respect to v_q/u_q. They may also be obtained by minimizing the ground state energy implicit in Eq. (32).

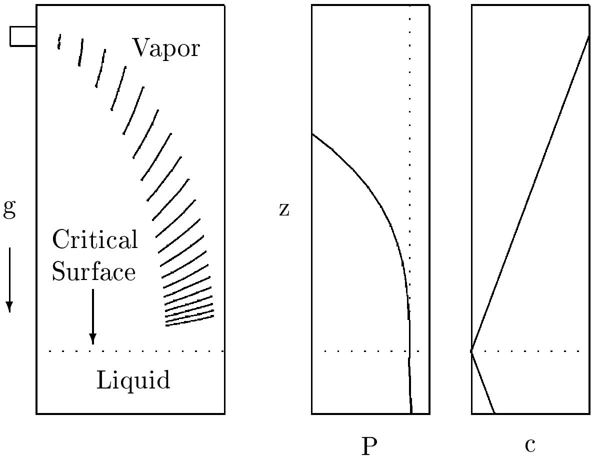

FIG. 6: Thought experiment in which a tank in earth's gravity is filled with cold superfluid characterized by a critical equation of state. The pressure increases toward the bottom of the tank and eventually reaches, and surpasses, the critical value. Sound quanta generated by a transducer are refracted toward this "horizon" and stall there, just as light does near a classical Schwarzschild event horizon. On the right are the pressure profile given by Eq. (37) and the sound velocity profile given by Eq. (38).

VII. CRITICAL EVENT HORIZON

Let us now turn to the black hole paradox. Rather than trying to resolve the problem by promoting a specific theory of gravity, which is probably not falsifiable anyway, let us use our understanding of quantum fluids to establish a simple point: Zero-temperature phase transitions generate the same kinds of apparent illogic one finds at black hole surfaces. This result is very general and thus also applies to candidate microscopic theories of gravity.

Imagine a thought experiment, illustrated in Fig. 6, in which a tall tank on the surface of the earth is filled with a zero-temperature quantum fluid described by a critical equation of state - for example, Eq. (16) with $d = 8a/27b$. The pressure increases toward the bottom of the tank due to gravity and, at some critical depth, reaches, and then surpasses, the critical pressure $p_c = a/27b^2$. If we now stimulate the system near the critical surface with a sound transducer, the injected quanta will be attracted by the surface because the propagation speed is lower there. This is the exactly same effect as refraction of light toward the center of a lens or of ocean waves toward a beach—or the gravitational attraction of light by a black hole. For both the critical surface and the black hole horizon this speed actually vanishes, causing the waves to stall and never reach the surface in finite time. However, this paradox has a simple resolution in the case of the fluid: The coherence length ξ diverges to infinity as one approaches the horizon, and the laws of quantum hydrodynamics fail. The waves cease to have all meaning as compressional sound and begin doing things disallowed by hydrodynamics, such as decay and thermalize.

Let us now consider the possibility that this analogy might be *apt* rather than just interesting. Pathologies in the ultraviolet are one of the central problems of general relativity. We presently have no way to resolve them, and the difficulties are so severe that there is serious talk about a need to change the laws of quantum mechanics[23]. Moreover the existing experimental record says nothing about this matter because it does not extend to the Planck scale. It certainly does not preclude possibility that relativity might simply fail at the event horizon through the divergence of a coherence length. Since distinguishing an emergent phenomenon from a fundamental one at long wavelengths is impossible, this amounts to a serious logical flaw in the way we normally think about relativity. It obligates us to take seriously the possibility that black hole horizons may be phase transitions of the vacuum of space-time, that they are described inaccurately by the Einstein field equations in the same way that critical surfaces are inaccurately described by quantum hydrodynamics, and that elevation of relativity to a position of transcendence is the source of the entire problem. Were this the case, it would instantly resolve all incompatibilities between general relativity and quantum mechanics, including the unitarity of the scattering matrix and the loss-of-information paradox.

The analogy between the critical surface and the black hole horizon may be made quite precise. In terms of the critical density $\rho_c = M(3b)^{-1/2}$ we have for the equation of state near criticality

$$\frac{p}{p_c} - 1 = 12(\frac{\rho}{\rho_c} - 1)^3 \quad . \tag{35}$$

Since the force of gravity is just a device for achieving a density increase, we are allowed to weaken gravity at the critical surface according to the rule

$$g = g_0(1 - e^{-z^2/\ell^2}) \quad , \tag{36}$$

in order to improve the black hole analogy. We then have

$$\frac{p}{p_c} \simeq 1 - \frac{g_0}{2\ell^2 c_0^2} z^3 \quad , \tag{37}$$

where $c_0 = \sqrt{p_c/\rho_c}$ is a velocity scale, and

$$c \simeq (\frac{6c_0 g_0}{\ell^2})^{1/3} |z| = \frac{|z|}{\tau} \quad . \tag{38}$$

This is precisely the rule with which the speed of light, measured with a clock at infinity, vanishes at the event horizon of a Schwarzschild black hole. Classical hydrodynamics predicts that small density fluctuations $\delta\rho$ propagate near the critical surface according as

$$\nabla \cdot [c^2 \nabla(\delta\rho)] = \frac{\partial^2(\delta\rho)}{\partial t^2} \quad . \tag{39}$$

This is not significantly different from the scalar wave equation

$$\nabla \cdot [c \nabla \phi] = \frac{1}{c} \frac{\partial^2 \phi}{\partial t^2} \tag{40}$$

one obtains from

$$\frac{\partial}{\partial x^\mu} (\sqrt{-g} \, g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu}) = 0 \tag{41}$$

using the gravitational metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dx^2 + dy^2 + dz^2 - c^2 dt^2 \; . \tag{42}$$

Failure by means of a diverging coherence length is so subtle that there is a precise sense in which the failure does not occur at all. The issue is an order of limits. Suppose we play a game in which you first declare how close to the critical surface you wish to go, then I look for a frequency at which hydrodynamics works all the way down to this height. If we do things in this order then I always win, since I can always pick the frequency sufficiently low that $\omega \xi / c << 1$ on your surface. If we do a proper experiment, on the other hand, in which you first fix the frequency and then I search for the height at which hydrodynamics fails, then I always lose. Thus if black hole horizons were like critical surfaces then there would be a precise sense in which general relativity was exactly true in all regions of space-time accessible to us. It would also be highly misleading. Our game shows in a physical way that knowledge of the classical field theory emerging in the $q, \omega \rightarrow 0$ limit is *not* sufficient for predicting all low-energy things near a phase boundary. We must also understand the ultraviolet behavior. An improper regularization—which is what this improper order of limits amounts to—can lead to physical nonsense.

VIII. EXPERIMENTAL SIGNATURES

Let us now discuss some experimental properties of the quantum fluid and its critical surface that might have analogs in quantum gravity and thus lead to observational tests of these ideas on real black holes.

A. Hydrodynamic Failure at High Energies

The failure of hydrodynamics at the critical surface is a specific case of a more general effect of ultraviolet breakdown. Since hydrodynamics is emergent it must fail in a measurement of the "vacuum" far from the critical surface, either through a deviation from relation $\omega = cq$ or an otherwise disallowed decay, at some characteristic scale ξ. The properties at this scale must evolve adiabatically to lower energies as the critical surface is approached, and eventually evolve into the critical properties.

B. Transparency

Let us return now to Fig. 6 and consider what happens to the acoustic energy beamed into the critical region. Part of the answer is suggested by Fig. 5, which shows that a sound quantum penetrating into the critical region might morph adiabatically into a free boson, traverse the region in finite time, and emerge intact from the other side. To quantify this effect, however, we must evaluate the rate at which a boson in the critical region decays. This may be thought of either as an acoustic nonlinearity or knocking extra bosons out of the condensate. At the critical point the lagrangian is effectively

$$\mathcal{L} = \psi^* (i\hbar \frac{\partial}{\partial t} + \mu) \psi - \frac{\hbar^2}{2M} |\nabla \psi|^2 + 3 p_c v_c^2 (|\psi|^2 - \psi_0^2)^4 . \tag{43}$$

The corresponding quantum hamiltonian is

$$\mathcal{H} = \sum_{\mathbf{q}} \frac{\hbar^2 q^2}{2M} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + \frac{3 p_c v_c^2}{L^3} \sum_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3 \mathbf{q}_4}$$
$$\times \delta(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4) (a_{\mathbf{q}_1} + a_{-\mathbf{q}_1}^\dagger)(a_{\mathbf{q}_2} + a_{-\mathbf{q}_2}^\dagger)$$
$$\times \, (a_{\mathbf{q}_3} + a_{-\mathbf{q}_3}^\dagger)(a_{\mathbf{q}_4} + a_{-\mathbf{q}_4}^\dagger) \quad . \tag{44}$$

The extremely high order of the nonlinearity means that the fastest decay process is emission of two extra bosons. The contribution of this process to the imaginary part of the boson self-energy is

$$\mathrm{Im}\Sigma_{\mathbf{q}}(\omega) = 12 (\frac{3 p_c v_c^2}{L^3})^2 \sum_{\mathbf{q}_1 \mathbf{q}_2} \mathrm{Im} \Bigg[\hbar\omega$$
$$- \frac{\hbar^2}{2M} (|\mathbf{q}_1|^2 + |\mathbf{q}_2|^2 + |\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}|^2) + i\epsilon \Bigg]^{-1}$$
$$= - \frac{3\sqrt{3}}{4\pi^2} (\frac{M}{\hbar^2})^3 (p_c v_c^2)^2 (\hbar\omega - \frac{\hbar^2 q^2}{6M})^2 \Theta(\hbar\omega - \frac{\hbar^2 q^2}{6M}) . \tag{45}$$

Thus the decay rate for a boson of energy $\hbar\omega = \hbar^2 q^2 / 2M$ is

$$\frac{\hbar}{\tau_{\rm rad}} = \frac{2}{\sqrt{3}\pi^2} (\frac{M}{\hbar^2})^3 (p_c v_c^2)^2 (\hbar\omega)^2 \quad . \tag{46}$$

This implies that the free boson becomes more and more sharply defined as its energy is lowered, so that in the low-energy limit one retrieves the ideal noninteracting bose gas[24].

The condition for transparency is $\tau < \tau_{\rm rad}$, where τ is defined by Eq. (38). The reason is that the time it takes the boson to traverse the critical region is always τ, regardless of its energy. The width of the critical region grows with the boson's momentum as $\hbar q \tau / M$, but the velocity also grows as $\hbar q / M$, and the two effects cancel.

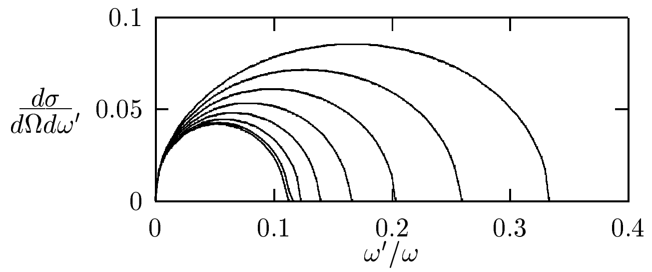

FIG. 7: Differential cross-section for prompt fluorescence defined by Eq. (47). The bosons are assumed to be normally incident at frequency ω. The various curves are different values of θ ranging from 0 to $\pi/2$. The maximum value of ω'/ω for radiation returned normally ($\theta = 0$) is 1/9.

C. Critical Opalescence

If the critical surface is at temperature T then there is a second decay process in which the boson scatters off of another boson thermally excited into the vacuum. This corresponds to conventional critical opalescence. It is straightforward to calculate, but we shall just state the result here. Up to factors of order 1, the rate as the same as in Eq. (46) except for the substitution $(\hbar\omega)^2 \rightarrow (k_B T)^{3/2}(\hbar\omega)^{1/2}$. Thus for frequenceis much larger than $k_B T/\hbar$ this process is irrelevant.

This has the interesting and important implication that the ratio $\tau_{\rm rad}/\tau$ determines whether the critical surface is optically thin or thick. When this parameter is much less than 1 the surface is very thin, and therefore not "black".

D. Prompt Fluorescence

There is a net probability of about 0.06 that one of the bosons created in a decay will escape back out of the surface. This fluorescence signal is prompt and has a characteristic spectral shape. Let us assume for simplicity that $\tau_{\rm rad} < \tau$, so that all the quanta impinging on the surface decay. In the opposite case one just reduces the signal by the decay fraction. Let us also assume that the bosons are normally incident at frequency ω. Then the differential cross-section per unit area A of the surface to scatter back into solid angle $d\Omega$ and between frequency ω' and $\omega' + d\omega'$ is

$$\frac{d\sigma}{d\Omega d\omega'} = \frac{27A}{16\pi^2\omega}\sqrt{3x[1 - 3x - 2\sqrt{x}\cos(\theta)]} \; , \qquad (47)$$

where $x = \omega'/\omega$ and θ is the polar angle of exit just above the critical region, i.e. before the signal is defocused by outward refraction. This result is plotted in Fig. 7.

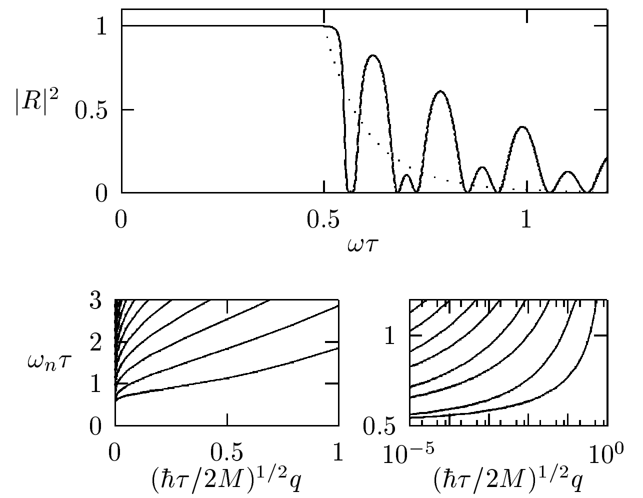

FIG. 8: Bottom: Resonant frequencies ω_n obtained by solving Eq (48) for various values of momentum q in the interfacial plane. Top: Reflectivity as a function of frequency showing structure at ω_n. The precise shape of this curve depends on how far above the surface the law $c = |z|/\tau$ remains valid. The dotted curve is a plot of Eq. (50).

E. Reflection Resonances

When $\tau_{\rm rad} >> \tau$ the bosons with parallel momentum q form bound states in the plane of the critical surface. This result is obtained by solving the wave equation

$$\hbar^2 \frac{\partial^2\psi}{\partial t^2} = (\frac{\hbar}{\tau})^2 \nabla \cdot (z^2 \nabla\psi) - (\frac{\hbar^2}{2M})^2 \nabla^4 \psi \qquad (48)$$

assuming frequency ω and momentum $\hbar q$ in the plane. The resulting eigenfrequencies ω_n are shown in Fig. 8 as a function of q. When $q >> (2M/\hbar\tau)^{1/2}$ the frequencies have the simple harmonic oscillator form

$$\hbar\omega_n \simeq \frac{\hbar^2 q^2}{2M} + (n + \frac{1}{2})\sqrt{2}\,\frac{\hbar}{\tau} \quad . \qquad (49)$$

This limit is difficult to achieve, for this is the condition implicit in Eq. (46) for $\tau_{\rm rad} < \tau$. When q is very small, on the other hand, the resonances converge together slowly and, at $q \rightarrow 0$, collapse to a continuum characterized by the reflectivity

$$|R|^2 = \begin{bmatrix} 1 & \omega\tau < 1/2 \\ \cosh^{-2}(\pi\sqrt{(\omega\tau)^2 - 1/4} & \omega\tau > 1/2 \end{bmatrix} \qquad (50)$$

This is plotted in Fig. 8. Also plotted is a sketch of the kind of reflection signal that this effect would tend to generate - a transmission resonance at every "bound" state. More than a sketch is unfortunately not possible because the details of the spectrum depend on how far above the surface the relation $c = |z|/\tau$ remains valid.

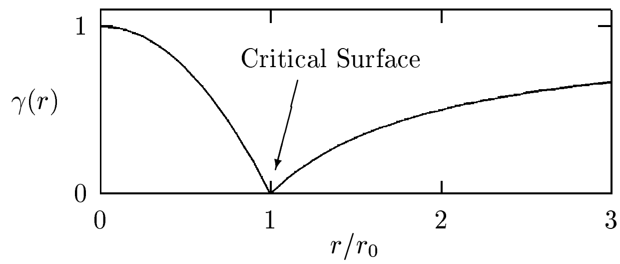

FIG. 9: Proposed solution for interior of black hole described by Eqs. (52) and (54). Note that the speed of light, measured by clocks at infinity, rises linearly on either side of the horizon, just as the sound speed does in Fig. 6.

F. Heat Capacity

The heat capacity of the critical surface is finite. Assuming Eq. (22) with $c = |z|/\tau$, we have for the thermal energy per unit area

$$\frac{E}{L^2} = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} dz \int_0^\infty q^2 dq \frac{\hbar\omega_q}{\exp(\beta\hbar\omega_q) - 1}$$

$$= \frac{\zeta(3)}{\pi} (\frac{k_B T}{\hbar})^3 M\tau \quad , \tag{51}$$

where $\zeta(3) = 1.202....$

IX. DE SITTER INTERIOR

In order to talk experimentally about real black holes it is now necessary for me to speculate about what is inside them. This is extremely dangerous since, as we have discussed, it is fundamentally impossible to infer the nature of one phase from low-energy measurements made on another. However, in order to stimulate thinking on what kinds of measurement one could profitably do on the black hole itself it is necessary to be concrete, and this requires that we specify what happens when one crosses the horizon.

I shall presume that that the analogy with Fig. 6 is literally correct. Catastrophic jumps of the metric characteristic of a first-order transition cannot be ruled out on any logical basis, nor can a non-relativistic equation of state inside the black hole. However, a continuous transition does the least violence to classical general relativity and allows the Einstein field equations to be valid, in the limited sense we have described, everywhere experimentally accessible to us. If the analogy is apt then the relevant emergent principle - in this case relativity - must be restored on the other side of the surface in a mirror-symmetric way. This means that the metric must exist on the other side, satisfy the Einstein field equations, and be characterized by a speed of light, measured by clocks at infinity, that *increases* with distance from the horizon, just as the sound speed does on the other side of the critical surface. As shown in Fig. 9, this does not leave one much flexibility. The boundary conditions at $r = 0$ require the black hole to contain positive energy density but negative pressure. This is the characteristic feature of de Sitter space. Indeed for the the specific metric

$$ds^2 = \gamma(r) dr^2 + r^2 d\Omega^2 - c^2 \gamma(r) dt^2 \quad , \tag{52}$$

the choice of cosmological-constant matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \left[\begin{array}{cc} 3r_s^{-2} g_{\mu\nu} & r < r_s \\ 0 & r > r_s \end{array} \right] \quad , \tag{53}$$

where $r_s = 2G\mathcal{M}/c^2$, with $\mathcal{M}$ the black hole mass, works nicely, and we obtain

$$\gamma(r) = \left[\begin{array}{cc} 1 - (r/r_s)^2 & r < r_s \\ 1 - r_s/r & r > r_s \end{array} \right] \tag{54}$$

This leads to the interesting idea that the two phases might be distinguished by the values of their cosmological constants.

An important experimental consequence of this solution is that there *would* be a local measurement capable of detecting proximity to a black hole. Black hole horizons cannot be phase transitions unless the relativity principle itself is emergent. Einstein gravity is based on very little other than the principles of relativity and equivalence, so if both of these are true then the predictions of classical general relativity must also be true—notably that black holes must form in the conventional way and *not* be analogous to the critical surface. Thus relativity would have to fail at sufficiently high energy scales whether one were near the black hole or not. The energy scale of this failure—the ultraviolet cutoff of the theory—would be both measurable and position-dependent. Its lowering to zero would signal proximity to the event horizon.

A reasonable guess for the scale at which the principles of emergence should fail is the Planck length $\xi_p = (\hbar G/c^3)^{1/2} = 1.61 \times 10^{-33}$ cm. However, here one must be cautious. The superfluid analog of Newton's constant is the inverse mass density ρ^{-1}. This is determined by adding the term

$$\delta\mathcal{L} = V_2 a^3 \Big[|\psi(\mathbf{r})|^2 + |\psi(-\mathbf{r})|^2 \Big] \tag{55}$$

to the lagrangian of Eq. (11), and then inducing acoustic radiation by moving the parameter $\mathbf{r}(t)$ in a circular orbit of radius $\ell/2$ at frequency ω. After some algebra one finds the power radiated to be

$$\mathcal{P} = \frac{1}{5\pi\rho c^5} \mathcal{M}^2 \ell^4 \omega^8 \quad , \tag{56}$$

where $\mathcal{M}$ is the mass accumulated around $\mathbf{r}$ by the potential. The corresponding quantity in Einstein gravity is

$$\mathcal{P} = \frac{2}{15} \frac{G}{c^5} \mathcal{M}^2 \ell^4 \omega^6 \quad . \tag{57}$$

The disparity in the powers of ω comes from the fact that hydrodynamics is a monopolar theory while gravity is quadrupolar. Combining the mass density and the sound speed dimensionally into a length, one obtains $(Mc/\hbar v)^{1/4} = \eta^{1/4}\xi$, where $\eta = \xi^3/v$. Unfortunately the dimensionless constant η cannot be measured in any $q, \omega \to 0$ experiment. If $\eta \simeq 1$, as is the case in superfluid ^{4}He, then dimensional analysis gives a reasonable estimate of ξ. Otherwise it does not.

This model suggests that cosmological black holes may be optically thin, and thus not "black" at all. For a solar-mass black hole ($\mathcal{M} = 2 \times 10^{33}$ gm) we have [*cf.* Eq. (38)] $r_s = 3.0$ km and $\tau = 2r_s/c = 2.0 \times 10^{-5}$ sec. To estimate $\tau_{\rm rad}$ let us assume that the vacuum far from the black hole is analogous to the quiescent fluid, and that $\eta = 1$ there, so that the coherence length ξ can be determined by dimensional analysis and thus equals the Planck length. Let us further assume that the equivalent fluid density and pressure are not far from their values at the critical point, so that $p_c v_c$ is Planck energy $M_p c^2 = \hbar c/\xi$ and M is the Planck mass M_p. Then we have

$$\tau_{\rm rad} = \frac{\sqrt{3}\pi^2 c}{\xi_p \omega^2} = \frac{3.2 \times 10^{44} \; {\rm sec}^{-1}}{\omega^2} \quad , \tag{58}$$

where ω is the frequency far away from the black hole. Thus the horizon would be transparent to gravitons (and presumably any other particle) of energy less than $\hbar\omega_{\rm max} = 2.6 \times 10^9$ eV. Were this the case, the black hole would look like a powerful defocusing lens.

The most important difference between the model of Fig. 9 and a traditional black hole is its finite specific heat. From Eq. (51) we find that the total thermal energy contained at the horizon of a black hole at temperature T, measured at infinity, is

$$E = 8\zeta(3) (\frac{r_s \; k_B T}{\hbar})^3 \; \frac{M}{c} \quad . \tag{59}$$

It is absolutely clear that the cold quantum critical surface does not radiate, since it is in its ground state, and it is also clear that the surface may be raised to arbitrary temperatures by adding heat. Thus this analogy is fundamentally at odds with Hawking's prediction that a black hole should emit thermal radiation with a temperature proportional to its mass[25]. Unruh[26] showed a number of years ago that traditional Hawking radiation is emitted from caustic surfaces of transsonic superfluid flows, and Jacobson and Volovik[27] have recently made a good case that this also occurs at "superluminal" solitonic domain walls of superfluids. The long-wavelength description of these systems is identical to that of the critical surface discussed here, but the ultraviolet description is different. Since Hawking's regularization procedure has no microscopic justification there is reason for concern that his result may be an artifact of fictitious motion encoded in the cutoff procedure. The heat capacity implicit in Eq. (59) is large. The temperature at which this energy equals $\mathcal{M}c^2$ for a solar-mass black hole is $k_B T = 141$ eV.

The event horizon in this model contains a large zero-temperature stress, or negative surface tension, like that in a steel pressure vessel, holding back the negative pressure of the cosmological-constant matter on the inside. This, however, is arguably a symptom of the breakdown of relativity and not physically meaningful stress. It is not detectable in any measurement performed on the outside other than the the fluorescence and reflection structure shown in Figs. 7 and 8, nor does it exist in any region of space-time where Einstein gravity is valid (on length scales longer than ξ). It resides only on an infinitely thin surface at which ξ has diverged to infinity and neither the metric nor the curvature tensor is defined on any scale. If one insists on thinking of this stress conventionally then it is large. For any choice of $\gamma(r)$, conservation of momentum requires the radial pressure jump across the surface to satisfy

$$\Delta T_r^r = \frac{1}{r_0} \int_{r_0 - \epsilon}^{r_0 + \epsilon} (T_\theta^\theta + T_\phi^\phi - 2T_r^r) dr$$

$$= \frac{1}{2r_0^2} \int_{-\sqrt{r_0 \epsilon}}^{\sqrt{r_0 \epsilon}} (T_\theta^\theta + T_\phi^\phi - 2T_z^z)|z| dz = \frac{3}{4\pi} \; \frac{\mathcal{M}}{r_s^3} \quad . \tag{60}$$

The thermal contributions to the pressure integral become comparable to this only when the total thermal energy approaches $\mathcal{M}c^2$. Denoting the these by δT_μ^μ we have

$$\frac{1}{2r_0^2} \int (\delta T_\theta^\theta + \delta T_\phi^\phi)|z| dz = \frac{\zeta(3)}{\pi} M (\frac{k_B T}{\hbar c})^3 \quad , \tag{61}$$

following Eq. (51).

X. CONCLUSION

My lecture today has been intentionally iconoclastic, and I hope you will all take it in the spirit of fun and as a starting point for reflection about the gravity problem in new ways. It has been my experience that good theoretical physics is empowering, in that it enables thinking to take place that would otherwise not occur, and, in its highest form, facilitates experiments that would otherwise not be done. This is a difficult and often dangerous task, as we are paid to be technicians, not visionaries, and can be just as severely punished for political incorrectness as a governor or congressman. However, this activity is the most important thing we do, and perhaps even the *only* important thing we do, for experimentalists are usually smart enough to model for themselves but cannot take expensive risks without help. For those of you younger than I am and feeling a bit unsure about how this all works, let me assure you of one of the great truths of our discipline: Experimentalists are amazing and wonderful people. They have clever tricks you or I could never guess and are always on the prowl for something to earn them glory. Communicating important ideas to them in a clear and courageous way is the best way I know both to earn one's keep and to generate science that lasts.

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