PUTTING A SPIN ON THE AHARONOV-BOHM OSCILLATIONS

Jeeva Anandan

Department of Physics and Astronomy

University of South Carolina

Columbia, SC 29208, USA

and

Clarendon Laboratory

University of Oxford

Parks Road

Oxford, UK

E-mail: jeeva@sc.edu

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Of the many fascinating consequences of quantum mechanics, among the more mysterious ones is the Aharonov-Bohm effect. According to classical physics, a charged particle would be influenced by a magnetic field only if the particle goes through a region in which the magnetic field strength is nonzero. But according to quantum mechanics, if the quantum wave representing the state of a charged particle, such as an electron, is split into two waves that go around a solenoid and interfere, the resulting interference pattern is influenced by the magnetic flux enclosed by the waves, even though the particle is nowhere in the region of non vanishing field strength. This was predicted by Aharonov and Bohm in one of the most influential papers of physics in the latter part of the twentieth century [1]. In addition to its charge, the electron has a magnetic moment proportional to its spin. So, if the magnetic field strength is non vanishing along the electron wave, then in addition to the above flux dependent effect, there are also spin dependent effects due to the interaction of the magnetic moment with the magnetic field. If in addition an electric field is present, there is the so the called spin-orbit interaction of the magnetic moment with the electric field [2], which further influences the interference pattern [3]. All these effects are present in a recent beautiful experiment performed by Yau, De Poortere and Shayegan [4, 5].

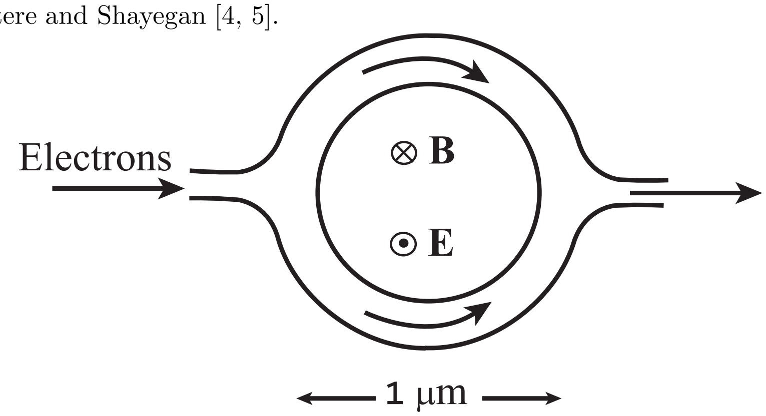

Figure 1. **Electrons enter and leave the ring structure as indicated by the horizontal arrows. The quantum wave associated with each electron in the entrance region splits into two wave packets that go around the ring, as indicated by the curved arrows, and interfere in the exit region. Homogeneous magnetic field B and electric field E are applied normal to the plane of the ring. Whether the interference is constructive or destructive, and correspondingly whether the current is maximum or minimum, depends on the values of B and E. Hence, varying these fields leads to oscillations of the current.**

An interesting aspect of this experiment is that it is a miniaturized version of the original experiments that confirmed the Aharonov-Bohm effect [6]. The apparatus consists of a ring structure (Fig. 1) with a diameter of only about one μm (micro-metre or one- thousandth of a millimetre) that is fabricated inside a GaAs/AlGaAs heterostructure. This is an example of the new field of mesoscopic physics that deals with structures that are intermediate between atomic and macroscopic scales. The linear dimensions of the tiny apparatus used in mesoscopic physics vary from about a μm to a nano-metre, which is only about ten times the size of an atom. If such a tiny apparatus made of metals or semi-conductors is made sufficiently cold (about 30 milli-kelvin) then the conduction electron waves are coherent over the entire apparatus, because there is no randomization due to inelastic scattering. This leads to interesting quantum effects. In the present experiment, the conduction electrons are like a quantum gas that enters and exits the ring (Fig. 1), and therefore constitutes a current. Homogeneous magnetic and electric fields are applied normal to the plane of the ring structure. The electric field $\mathbf{E}$ is needed to confine the electrons to this plane.

It was predicted [7] that the electrical resistance of the current through a mesoscopic ring would vary as the magnetic flux Φ through the ring is varied by changing the strength of the magnetic field $\mathbf{B}$. Moreover, this magneto-resistance is an oscillatory function of the magnetic flux with period h/e, where h is Planck's constant and e is the charge of the electron. This is a consequence of the fact that the Aharonov-Bohm phase shift is $\frac{2\pi e}{h}\Phi$, and the current is maximum when there is constructive interference and minimum when there is destructive interference, for a given externally applied potential. This has been well confirmed by experiments [8], including the present one (Fig. 2). This means that if one writes the magneto-resistance function $R(\Phi)$ as a sum of all possible (normalized) periodic functions of Φ, then the coefficients of this expansion that multiply the functions whose period τ is equal to or very close to h/e are very large compared to the other coefficients. In other words, the Fourier transform of R that is the function $\tilde{R}(f)$ of the frequency $f = 1/\tau$ (whose values are the coefficients of the above expansion) would be highly peaked at $f = e/h$.

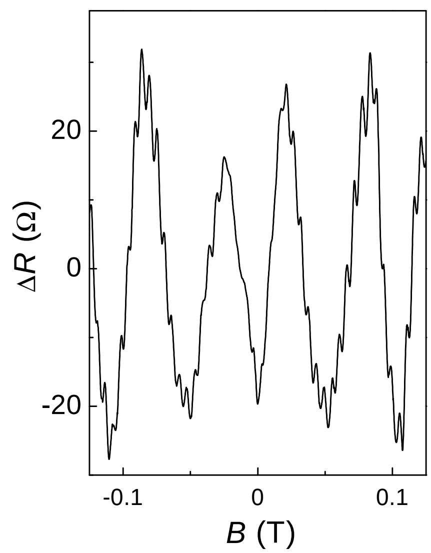

Figure 2. **ΔR is the resistance (in Ohms), after subtracting away a smooth background resistance, in the presence of an externally applied magnetic field B (in Teslas). As B is varied (with the externally applied electric field fixed), ΔR oscillates. This is due to the oscillation of the current through the ring structure in Fig. 1 that is inversely proportional to the resistance, for a constant applied potential difference, according to Ohm's law.** (This graph is taken from ref. 4.)

In the experiment of Yau et al [4], this peak was observed. But in addition two other smaller peaks on either side of the main peak were also observed, suggesting a modulation of the Aharonov-Bohm oscillation of R. It is reasonable to regard these side peaks as being due to spin-dependent effects. These may be obtained by supposing that the electron interacts with an effective magnetic field [2] that is $\mathcal{B} = \mathbf{B} - \frac{1}{2}\mathbf{v} \times \mathbf{E}$, where $\mathbf{v}$ is the 'velocity' of the electron in the approximation that its wave is approximately a plane wave. The factor $\frac{1}{2}$ distinguishes the electron from the corresponding effective magnetic field experienced by a neutral dipole, such as the neutron, in the combined magnetic and electric fields. For an electron in an atom, this difference is traditionally attributed to the semi-classical Thomas precession [2]. It should be noted that the $\mathbf{E}$ experienced by the electron is much larger than the applied electric field. This has been determined to be the case experimentally, and has been attributed to complex band effects in the semi-conductor [9].

The above combined effect of the electric and magnetic fields on the electrons may also be regarded as arising from Berry's phase [10]. This is a geometric phase acquired by the wave function when it evolves slowly (adiabatically) and returns to the original state. In the present case, because $\mathcal{B}$ varies slowly during the motion of the electron along each semi-circular ring, the components of the spin state of each electron in the direction of $\mathcal{B}$ remain pinned to this varying direction. However, owing to the rapid change in $\mathbf{v}$ at the point of entry, the initial directions of $\mathcal{B}$ are different for the two states that go around the semi-circular rings from this point and interfere at the point of exit. Hence, only the non-cyclic Berry phase corresponding to a spin state going around a semi-circle, which is then completed to a closed curve by the longitude, may be defined for each semi- circular ring. Berry's phase is half the solid angle subtended by the curve traced by $\mathcal{B}$ on the sphere that represents all possible directions of the $\mathcal{B}$ and completed into a closed curve by the geodesic joining the initial and final directions at the center of this sphere. For unpolarised electrons, as in the present experiment, the average of these Berry phase factors may be observed.

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